Real Estate Price Dynamics and the Value of Flexibility

By David Geltner* & Richard de Neufville**

PRELIMINARY
This version: June, 2017

Abstract:

This paper presents a model of real estate asset price dynamics based on empirical evidence, economic theory, and common sense. We note important differences between real estate versus stock market price dynamics. We then use this model to simulate the investment performance of archetypical property investments, in stabilized income-producing property, and in multi-asset development projects. We quantitatively explore the effects of uncertainty on investment performance, for example, the positive skew in the valuation distribution and the positive bias in the pro-forma return compared with the expected return due to Jensen's Inequality. We then focus on the value added by flexibility, including various types of real options in development projects. We consider the value of resale timing flexibility in stabilized property investment, and the value of production delay and product switching options in the development project. We find that with typical real estate price dynamics, flexibility adds considerable value, and greatly improves the expected return. In stabilized property investment, resale timing flexibility can add 25% to the private valuation of the property. In multi-asset development, we find that timing options are redundant within themselves, but additive with product type options. Delay options provide significant downside protection, and combined with product switching can add over one-third to the value of the project as measured by its implied bid-price for the land.

Acknowledgments:

The authors gratefully acknowledge the stimulation and ideas from the students over the years in our MIT class, "Tools for Analysis".

----

* MIT Center for Real Estate (dgeltner@mit.edu), Contact author.

** MIT Institute for Data, Systems & Society (ardent@mit.edu).

Real Estate Price Dynamics and the Value of Flexibility

By David Geltner & Richard de Neufville

1. Introduction & Background

Real estate investment decisions are made in a world of uncertainty, as economists understand a special meaning of this term. Uncertainty refers to the fact that relevant future outcomes are not just unknown as of the present (when investment decisions and valuations are made), but we do not even know for certain the probability distributions and stochastic processes that will govern the outcomes. Yet, to aid in building our understanding, and in practical decision making, academic and industry analysts often make specific and quantitative assumptions, implicitly or explicitly building and using models of the asset price dynamics that so importantly determine investment performance. In effect, we try to substitute what economists label as "risk", that is, known probabilities governing price dynamics, for "uncertainty", replacing "unknown unknowns" with "known unknowns". We do this using some combination of empirical evidence of price dynamics from history, plus assumptions based on economic theory, as well as basic logic and common sense. For example, historical evidence shows the prominence of a long-period cycle in commercial (investment) property prices in the United States, 15 to 20 years, and not necessarily echoing the general business cycle. Economic theory suggests that prices in a well-functioning asset market should display some component that is a random walk, as the arrival of relevant news, unpredictable information, moves prices in such a market. Logic and common sense tell us that prices of the unique, whole assets traded in the private property market must require some time to fully reflect the implication of news, leading to some degree of inertia, or auto-regression in returns, for no one can know the exact market value of any specific property at any given time.

Over the past generation, we have seen an ever-growing quantity and quality of commercial property price and valuation data, as well as advances in economic theory and econometric techniques. This has allowed a more sophisticated understanding and quantification of the price dynamics of investment property. In this paper, we use such knowledge to explore quantitatively the value of flexibility in property investment and development. Obviously, real estate investors and developers have various types of flexibility in management and decision making, such as when to sell an asset they own, or when to start construction on a development project. And clearly, flexibility is valuable in the face of uncertainty. Flexibility enables investors to avoid downside outcomes, and pursue upside opportunities. We posit a model of commercial property price dynamics that we employ in Monte Carlo simulations of property investment and development plans and decisions. We refer to this approach as "behavioral analog" modeling, because we employ simple, stylized option exercise decision rules that approximate typical decision behavior, rather than modeling equilibrium asset pricing in a formal economic framework. We apply the simulation analysis to archetypical real estate investment and development projects, allowing us to explore various types of flexibility. We conduct the analysis using simple spreadsheet software (Excel®). Combined with the behavioral analog nature of the model, based on traditional DCF valuation as widely practiced, we believe this approach makes our analysis transparent and accessible to non-specialized practitioners and decision makers.[1]

---

[1] Perhaps the major strand of literature in the urban and real estate economics field that relates to this subject is the real options literature, which is quite voluminous, beginning with Titman (1985), Capozza & Helsley (1989), and Williams (1991), followed by many others. Recent contributions include Grovenstein, Kau & Munneke (2011), and Clapp, Eichholtz & Lindenthal (2013), among others. The general conclusion from this literature is that optionality accounts for an important component of urban land value. Other fields of study have an even more extensive real options literature, including corporate finance, management science, forestry, and systems engineering. Influential texts include Dixit & Pindyck (1994), and Trigeorgis (1996). The present paper is distinguished from the prior literature primarily by our use of the transparent behavioral analog model using traditional DCF valuation based on

We find that flexibility adds an important amount of value to property investments and development projects. We can characterize this value in several respects. Resale timing flexibility for stabilized assets adds considerable value, but such value probably largely characterizes private valuations rather than market value, and the benefit is dependent on aspects of asset price dynamics that are specific to real estate as distinguished from stock market price dynamics. Flexibility in the construction phase of a typical large-scale multi-asset development project also adds important value. In such projects, various timing options, such as overall project start delay, and modular or phasing buildout delay, are valuable but substantially redundant among themselves, displaying rapidly declining marginal benefit. On the other hand, product switching (building type) options may often be of considerable value, and their value tends to be additive to that of the timing options, not redundant. The range and specificity of this type of exploration and discovery about the value of flexibility in a practical context, as well as the degree of realism in the underlying price dynamics assumptions, would be very difficult if not impossible to achieve using the traditional equilibrium modeling techniques of economics. And that traditional approach would likely lose the transparency that makes the results accessible to practitioners.

The remainder of this paper is divided into four sections. First, in Section 2 we discuss the price dynamics of investment property, and present our stylized assumptions that we use in our simulations. Section 3 explores the implications of resale timing flexibility in a typical stabilized income property investment. Section 4 introduces a typology of flexibility in

relatively realistic real estate price dynamics, together with the breadth of our analysis of various and multiple types of options and management flexibility in the specific context of real estate investment and development projects.

development projects and explores the value of various such options. Section 5 presents a brief overall conclusion.[2]

2. Investment Property Price Dynamics

Investment property refers to income-generating property, including apartments and commercial buildings. (We often use the term "commercial property" to refer to investment property, and in such use we generally mean to include private rental housing.) Price dynamics refers to how prices change over time. Clearly, price dynamics is a major determinant of the performance of real estate investments, and is particularly important in determining the nature and magnitude of risk faced by investors. Since we are focused on investment performance in the private property market, the type of price changes we are interested in is what may be referred to as "same property" price changes. That is, the price change of the investment asset experienced directly by the owner of (investor in) the property. This reflects any net depreciation in the structure value, that is, secular decline in property value due to aging and obsolescence of the structure even after and in spite of expenditures on routine capital improvements.[3]

As noted, we must quantify the uncertainty in future pricing in order to analyze investment risk quantitatively. We want to replace "uncertainty" with "risk" by making reasonable assumptions about the stochastic processes and dynamics that we can use to model the evolution of commercial property asset prices over time. We will employ such modeling in

---

[2] This paper is based on, and substantially reflects, Geltner & de Neufville (forthcoming).

[3] Thus, the type of price change we are focused on differs slightly from that used in national income accounting, which strives to reflect a constant "quantity" of property, by removing the effect of both age and capital improvements. (See Eurostat, 2017.) However, we do attempt to exclude value increments caused by major renovation, rehabilitation or redevelopment expenditures. We are focused on the property price path during the periods (typically decades) between such major capital infusions, the price experience of investment in "stabilized" property assets.

our simulation of investment performance that we use to analyze the value of flexibility in the later sections of this paper.

In this section we discuss real estate price dynamics in general, and the basis of the specific assumptions that we employ in the subsequent analysis. Our assumptions reflect a combination of historical empirical evidence, economic theory, and common sense. The specific evidence and assumptions apply to commercial property in the United States. However, much of the general discussion and considerations may apply also to housing or to other countries as well. We begin with a couple of threshold considerations.

Investment property assets are traded in the property asset market, but their value derives ultimately and fundamentally from their ability to produce income, which is generated in the space market. Thus, two separate and distinct markets are relevant. The space market is the rental market, with user/occupants (tenants) on the demand side and property owners on the supply side. The equilibrium in this market determines the rents and occupancy that governs the operating cash flows generated by the property assets. The asset market is where investors trade ownership of the income-generating assets, effectively a branch of the global capital market. The prices in the asset market, in effect the capitalization of the assets' future net operating cash flows, reflect the flow of financial capital (money) into and out of and within the asset market, as investors make their allocation decisions. Thus, property asset price dynamics are governed by actions, events, and phenomena in both the space market and the asset market. We need to consider both of these markets when thinking about property price dynamics. If rents in the space market rise 10 percent with other things equal, property prices may rise 10 percent. But if typical yields in the asset market rise from 10% to 11% with other things equal, property prices will fall by nearly 10 percent. The space market is not integrated across locations and building types.

Shopping centers in Denver do not compete for tenants against office buildings in New York. This allows for large, systematic cross-sectional dispersion in asset prices and price dynamics even though the asset market is relatively integrated, with investment money able to flow relatively freely between types and locations of property. Studies show that, similar to findings in the stock and bond markets, much of the longitudinal dispersion in property asset prices reflects changes in the asset market in addition to the space market, that is, asset price movements do not just reflect the operating fundamentals of the assets.[4]

In addition to the effect of the two markets, space and asset, another dichotomy is important to keep in mind in considering real estate price dynamics. This is the fact that property assets consist of two components which together create the value of the asset, the land (site and location), and the building structure. Evidence suggests that in the U.S. on average about half of the value of the average-age property (that is, with an average-age building on it) reflects the land value, and about half is the structure value.[5] These two components of value may reflect different types of price dynamics. Locations are unique, and this causes the supply of land to be relatively inelastic (completely inelastic for any given property). Land value therefore moves very sensitively with changes in demand for built property, and there are relatively weak constraints holding back the land component of the property value from following a Random Walk type of stochastic process.[6] On the other hand, the building is a produced good, in this respect not greatly different from other long-lived capital goods (such as motor vehicles, aircraft, ships, machinery, etc.). Basic micro-economic theory suggests that price movements reflecting

---

[4] See Shiller (1981), Geltner & Mei (1995), and Plazzi, Torous & Valkanov (2010).
[5] See Bokhari & Geltner, 2018.
[6] Ultimately, one imagines it would be difficult for the usage demand for the location to drift infinitely without bound. But in practical terms land value may be more closely modeled by a Random Walk than is the structure value component of property value.

the structure component of the property value should tend to revert toward long-run marginal production costs in the construction industry (exclusive of land costs). Construction exhibits rather elastic supply, hence, relatively little price volatility.[7]

In addition to theoretical considerations (including but not limited to the above), we can examine by now a rich set of empirical historical longitudinal data on investment property price and valuation indexes, as well as individual property prices and valuations. The major data sources that underlie the assumptions we employ in this paper include the National Council of Real Estate Investment Fiduciaries (NCREIF), and Real Capital Analytics Inc. (RCA). The former has been compiling appraisal-based and other data, and producing price and investment performance indexes, with data starting in 1978 (and related sources back to 1969). The NCREIF population of properties consists primarily of relatively large, prime properties held in the core investment portfolios of pension and endowment funds. The RCA database is much broader, essentially tracking all property sales over $2.5 million, but only since 2000. There are also a few ad hoc very long time series of real estate prices, such as that of the Herengracht in Amsterdam.[8] Other empirical indications relevant for investment property price dynamics come from the stock market, with the tracking of the equity share prices of publicly-traded Real Estate Investment Trusts (REITs). While REITs are levered and exhibit stock market price dynamics, not private real estate asset price dynamics directly, they also provide some insight about commercial property values, since such property composes nearly all of the assets of traded equity REITs.

Based on the above, we distinguish eight major potential elements within, and in some sense sources of, property price dynamics:

---

[7] For example, see various US Bureau of Labor Statistics price indices. Also see Gyourko & Saiz (2006), and Wheaton & Simonton (2007).
[8] See Chapter 5 in Geltner *et al* (2014).

1. Long-term trend
2. Random walk volatility
3. Cyclicality
4. Mean-reversion
5. Inertia (Auto-regression)
6. Price dispersion (noise)
7. Idiosyncratic drift
8. Black swans

We will discuss each of these in turn, and in the process will address some basic, general considerations for thinking about real estate price dynamics, as well as our specific assumptions that we employ in the subsequent simulation analysis.

1. Long-term Trend. The long-term trend is a per annum rate that in our simulations applies as a constant geometric growth rate throughout any given future scenario (any individual Monte Carlo "trial"). It reflects the long-term secular historical trend in investment property prices. As such, the long-term trend rate reflects the long-term trend in land value, plus the effect of the net deprecation of the building structure. As noted, on average in the US, land and structure value are about equal shares of property asset value.

Since land is overall in fixed supply, one might expect the long-run trend in nominal land value to grow slightly on average, perhaps at a rate similar to that of the nominal GDP (and recall that land on average is about half of commercial property value). However, investment properties tend to be located in central places, and the relative location value of central places in urban areas has been experiencing a long-standing secular decline relative to peripheral places. This reflects the flattening of the rent gradient caused by improvements in transportation and communication infrastructure and technology. Net depreciation reduces the real value of structures at a rate on average slightly over three percent per year of age. While the result varies across metro areas, Bokhari and Geltner (2016) report average commercial property value net depreciation in real terms of one to two percent per year of age as a fraction of total property value including land. As a result, in the United States over at least the past half century, the long-run trend in commercial property prices shows notably less nominal growth than that of single-family houses, and in fact has not quite kept up with inflation.[9] For commercial properties on average, increase in real land value, if any, has been insufficient to compensate for net depreciation in the structure.

In our simulation analyses in this paper we assume a long-term net growth trend of two percent per year for built property assets including land, and we simulate +/- 0.5%/year uncertainty range around that rate. It must be recognized that the US is an economically mature, land-rich country. Different assumptions might be appropriate in some other countries.

2. Random Walk Volatility. Volatility refers to the longitudinal standard deviation in returns, or price changes as a fraction of the price level. The volatility component referred to here is the component of asset price dynamics that reflects the arrival of news, "innovations" in the stochastic process governing property prices. As famously demonstrated by Samuelson (1965), to the extent that asset prices fully reflect all currently available information relevant to their value, the only thing that can move prices off their current values is the arrival of new information. New information is, by definition, unpredictable. Hence, the pricing process is "memoryless", that is, changes in any given period are uncorrelated with changes in past periods, resulting in the "Random Walk" type stochastic process. This type of volatility *accumulates* in the price levels, in the sense that the current price reflects all of the past news. Though pure Random Walk volatility produces returns that are uncorrelated across time, other elements of real

[9] See Geltner (2015).

estate price dynamics described in this section may produce either positive or negative auto-correlation, which may mask, though not eliminate the pure Random Walk volatility effect.

This component of property price dynamics refers to movements that are systematic across all individual assets, that is, aggregate market volatility. Idiosyncratic price movements may also reflect a Random Walk, as will be noted later in this section. The pure Random Walk component of property price movements may largely reflect the land value component of property value. In our simulations, we assume a Random Walk volatility (standard deviation in the stochastic innovations) of 10 percent per annum in the aggregate market (as in a relevant price index). This is based on a variety of direct and indirect empirical evidence.[10] We test for sensitivity between 5 percent and 15 percent. This does not comprise the total source of longitudinal standard deviation in property asset returns (the total volatility), as there are other types of dynamics (such as cyclicality and Black Swans), and there are idiosyncratic as well market sources of price changes and noise.

3. Cyclicality. This refers to a long-period, deterministic cycle in property asset price dynamics. Exhibit 1 shows the empirical evidence of a strong cycle of 15 to 20 years duration in commercial property in the United States over the past half century. It is remarkable how regular this cycle has been, and how it does not necessarily echo the overall economy business cycle. For example, there were recessions in 1980-81 and in 2001 that were not reflected in major or widespread falls in commercial property prices. On the other hand, down-cycles in the mid-1970s, early 1990s, and the Global Financial Crisis (GFC) of 2008-09 were associated with major crashes in commercial property. It is not certain what causes this cyclicality, and it is not

[10] See for example Geltner *et al* (2014), and Geltner (2015).

guaranteed that it will continue in the future, or with the same periodicity. But it would seem imprudent or unrealistic to ignore it in simulation analyses based on commercial property price dynamics. Hence, we model a deterministic sine-wave type cycle in our simulations, with period ranging randomly between 10 and 20 years across the simulation trials. Empirical evidence suggests that long-period cycles can occur in both the real estate space market and asset market, but the cyclicality need not be exactly the same.[11] In our simulations we model separate, but correlated, cycles in both markets. We model amplitude of +/- 25% (of mean level) in net rents in the space market, and +/- 200 basis-points of yield in the asset market, and we test for sensitivity to these assumptions. We model the cycle phase agnostically, allowing random equal probability for all phases across the simulated future histories (Monte Carlo trials).

4. Mean-reversion. This refers to the tendency of price levels to revert toward a long-run mean path. In our case, the long-run mean path is that represented by the long-term trend described above in item (1), that is, before including volatility or cyclicality. Mean-reversion (similar to the statistical concept of "error correction") is a phenomenon in the price levels, not in the returns *per se*. As noted, mean-reversion is expected in real estate prices due to the role of the built structure in the overall asset value. The value of produced goods tends to revert toward their replacement cost, minus depreciation. But mean-reversion is much weaker in real estate than in the pricing of most other long-lived capital goods, because of the land value component, and also because the structure, the produced good, is particularly long-lived.[12] In our simulations we assume a geometric mean-reversion rate of 30% per annum, meaning that only 30% of the deviation of the current price from the long-run mean price is reduced in a year. Even this effect

[11] See Geltner *et al* (2014), Chapters 2, 6, & 7; and Wheaton (1987, 1999).
[12] Bokhari & Geltner (2016) estimate an unconditional life expectancy of commercial structures in the US of 100 years.

is mitigated by the other price dynamics elements described in this section (such as volatility and cyclicality, and auto-regression). Mean-reversion tends to induce negative first-order autocorrelation into annual real estate returns.

5. Inertia (Auto-regression). Auto-regression refers to the tendency of price changes in one period to include a component of the price change in the previous period. Returns display positive serial correlation (positive first-order auto-correlation). This is caused by inertia, or "stickiness" in real estate asset pricing. This is related fundamentally to the fact that no one can know the exact market value of any given property at any given time. This is because whole, unique assets are traded in the property market (unlike the homogeneous common shares and securities traded in the stock and bond markets). Transaction prices result from private trades essentially between two parties, neither of whom knows the exact value of the asset. When news arrives that is relevant to value, the trades do reflect this news, but only gradually over time, as market participants gradually learn about movements in market values (largely through observation over time of prices in trades of "comparable" properties).

Appraisal-based indices, such as the NCREIF Property Index (NPI) display even more inertia and autocorrelation, because appraisals lag transaction price movements, in part because appraisers tend to anchor their valuations on prior appraisals. The NPI displays auto-regression with a first-order coefficient on the order of +0.7. But this is at the quarterly frequency, in which many appraisals are "stale" (not updated in the index). Annual-frequency auto-regression in the NPI is less than +0.5, and this certainly reflects more inertia than is in actual transaction prices. In our simulation analyses we model inertia with a first-order auto-regression coefficient of +0.2.

6. Price dispersion (Noise). This refers to the fact that consummated transaction prices tend to disperse around the unobservable "true" market value of the asset as of the time of the transaction. Again, this is because no one knows the market value of the asset. The buyer and seller negotiate (or there is a bid process), and the result is for practical purposes a random drawing from a distribution centered (by definition) on the market value. From the perspective of empirically based price indices, such dispersion appears like "observational error," random fluctuations in the index price level. Referred to as transaction price "noise", this imparts a negative first-order auto-correlation in price index returns, and can give a price index a spikey or "saw tooth" appearance. At some frequency this can appear similar to the effect of mean-reversion, as the negative auto-correlation in fact results from a type of "correction" of price level "error". At the individual property level, noise only occurs (but it always occurs) when the property is transacted. Thus, at the aggregate (price index) level, noise does not accumulate in the index level, and in this respect differs from Random Walk volatility (true pricing "innovations" as described in (2) above). We can get an idea of the magnitude of noise by examining the dispersion of transaction prices around recent appraised values in property populations such as that underlying the NPI.[13] In our simulations we have assumed a standard deviation of noise of five percent (of the true property value), and tested sensitivity ranging from zero (no noise) to 10 percent.

7. Idiosyncratic drift. This refers to true random increments in property value that may accumulate over time in the property price. Similar to the random dynamics described previously, only here we refer to increments that are unique to an individual property (or perhaps to a specific portfolio or group of properties, or a sub-market). This represents an additional source of volatility at the granular level. Another way to look at is that aggregate or market-level results as tracked by an index reflect some diversification effect, which reduces volatility. If, as

[13] See Geltner *et al* (2014), Chapter 12 (section 12.2.1, pp.265-267).

in our simulation analyses to follow, the focus is at a more granular level (of an individual investment or development project), then one must consider the additional volatility caused by idiosyncratic drift. It is a component of price dynamics that is uncorrelated with the aggregate market. Longitudinal changes in individual property values need not be completely determined by, or perfectly correlated with, aggregate market movements. For example, the market as a whole may be doing fine, but the anchor tenant in your property has just announced their intention to move.

We can get an idea of the typical magnitude of idiosyncratic drift, and also of transaction price noise, by analyzing the residuals from repeat-sales regressions used to produce repeat-sales transaction price indices, such as those of RCA. In other words, we examine the difference between individual property performance and the index performance.[14] In our simulations with multiple property types (to analyze the switching option in development projects), we assume idiosyncratic drift innovations with annual volatility of five percent, with a beta of unity and resulting contemporaneous cross-correlation between property types of approximately 80% (i.e., closely linked sub-markets, as for different types of houses or offices). We test for sensitivity ranging from zero to 10 percent idiosyncratic drift innovations volatility.

8. Black swans. This refers to the possibility of a discrete, large, negative event happening, rarely and unpredictably. Such events are sometimes described as "fat tails" events, referring to their magnitude and frequency being too great to be well described by the normal (Gaussian) probability distribution. The most famous recent example is the GFC of 2008-09, which seemed to "come out of left field," a seemingly surprise event that had great impact on

[14] See Yella (2016).

almost all capital assets. In our simulations, we model the possibility of "Black Swan" events as a Bernoulli event, that is, an event that has a certain probability of occurring in any given year. We model this probability as 5%, with a resulting 25% loss in property value which then mean-reverts back to normal at the 0.3 rate noted in (4) above. We constrain the Black Swan to occur at most only once in any 24-year simulated history (Monte Carlo trial). Thus, we model Black Swans as occurring on average once every 20 years, and the probability that a given scenario (trial) will experience a Black Swan event is: $1 - (0.95)^{23} = 69\%$.

* * *

The eight elements of real estate price dynamics described above are combined in our simulations of future possible real estate price paths, which we use in our Monte Carlo analyses of the effects of uncertainty and flexibility on investment performance. The price dynamics algorithm we use is described a bit more formally and technically in Exhibit 2. The result is independent, randomly generated, simulated future 24-year asset price histories that typically look like those pictured in the bottom panel of Exhibit 3. The Exhibit displays five randomly generated price histories, de-trended and indexed to a starting value of 1.00. The top panel of the Exhibit shows another set of five independent random price histories, only in this case depicting a nearly pure Random Walk, with 20% annual volatility. The top panel is a pretty good depiction of stock price dynamics, but not real estate price dynamics. The difference should be apparent visually (both panels are at the same scale). Real estate price dynamics are less purely random, and clearly display more cyclicality and mean-reversion tendencies. (In the Exhibit, the cycle is set deterministically to begin in the middle of the upswing in all five trials, but the period is random from 10 to 20 years.) Both sets of dynamics have similar annual volatility, approximately 20% (which in fact is typical for real estate individual assets and for blue chip stocks).[15] But the real estate returns have nearly 40% positive first-order auto-correlation, while the stock market returns are uncorrelated. And the real estate prices are clearly kept more within a range than are the stock prices. Real estate price dynamics are complex, and in performing simulation analysis there is value in what might be termed an "eyeball test." Do the simulated price histories *look* plausible? Do they look like investment properties could trace out price path histories that look like that? We think that the answer in the case of the bottom panel is "yes", while the answer in the case of the top panel of Exhibit 3 is "no".[16]

Finally, we should make a point about real estate price volatility and return frequency or return time horizons. In the case of the pure Random Walk price process, with zero autocorrelation in the returns, the longitudinal variance (square of the standard deviation or volatility) will be directly proportional to the time horizon over which the return is measured (inversely proportional to the frequency of the periodic returns series). For a given underlying volatility of innovations, the observed variance will be more than proportional to the horizon if the returns exhibit positive auto-correlation, and less than proportional if the returns exhibit negative auto-correlation. Thus, the effect of inertia and long-period cyclicality, which impart positive auto-correlation, is to increase the relative (or annualized) volatility, the longer the horizon (lower the return frequency of a series). The effect of cyclicality, mean-reversion and transaction noise is just the opposite, less than proportional variance, at least after some threshold return time horizon. It is therefore possible for the several types of price dynamics affecting real estate to mask each other. A series may appear with little auto-correlation, yet it actually exhibits

[15] In the real estate case, this volatility reflects not only the innovations in the Random Walk at both the market and asset-specific level, but also the effect of the other price dynamics, including the cyclicality, auto-regression, mean-reversion, Black Swans, and idiosyncratic transaction noise.
[16] Most of the formally rigorous economic real option models based on partial equilibrium criteria assume pure Random Walk price dynamics.

both inertia and mean-reversion. A series may appear smooth in the short run, yet it actually includes a component of Random Walk volatility. We know the eight effects described in this Section must exist in real estate price dynamics, based on economic theory and common sense, even if empirical evidence for them is weak.

3. The Value of Resale Timing Flexibility in Stabilized Asset Investment

Let us now apply the price dynamics described in the preceding Section in a Monte Carlo simulation to analyze the nature of uncertainty in a very basic type of real estate investment: a stabilized income-producing property asset (without leverage). Let us furthermore consider the effect on investment value and return performance of a ubiquitous type of flexibility, the ability to choose the time of disposition (resale) of the asset.

Begin by considering the "plain vanilla" investment property depicted in Exhibit 4. (Indeed, we will refer to this property as the "Plain Vanilla Building".) This is a slightly simplified and regularized representation of a stabilized investment property, with numerical values and relationships typical of mid-2010s in the US. Exhibit 4 depicts a 10-year pro-forma Discounted Cash Flow (DCF) investment projection and valuation, an essentially realistic version of micro-level (individual property) analysis typical of current practice in the industry. The property has an estimated market value ("MV") of $1000 (add however many zeros you want) reflecting a gross rent multiple of 10 (initial potential gross income of $100). The cap rate is 6 percent (defined by standard industry practice based on the next year's Net Operating Income of $60), and the net cash yield is 5 percent ($50 initial net cash flow, after routine capex). Rents and expenses, and indeed all cash flows, have a projected growth rate of 2 percent per year, which we will treat for our purposes as an unbiased, realistic projection. With only slight over-simplification, the yield is also assumed constant.[17] The Opportunity Cost of Capital (OCC), the market's expected total return on an unlevered investment in the asset, is assumed to be 7 percent per annum, measured as a round-trip going-in IRR, resulting in the $1000 present market value. The Plain Vanilla Building might be a rental apartment or an office building, or some other such typical stabilized income property asset.

Now suppose we subject this investment to the price dynamics described in the previous section, simulating 10 years of random future price realizations in both the space and asset markets. We center the future price probability distributions on the pro-forma projections, assuming that pro-forma projected cash flows and yields are indeed unbiased, the means of the relevant future probability distributions. We continue to assume, per the pro-forma, that the resale will occur at the end of the projected 10-year holding period, whether or not Year 10 turns out to be a good time to sell the property. In other words, we ignore the investor's flexibility to time the resale.

The result is the simulated investment performance ex post distributions shown in Exhibit 5, for the present value (at 7 percent discount rate) and the IRR (at the $1000 price).[18] Note that the mean of the ex post present value is indeed $1000, as the present value is a linear function of the cash flows.[19] However, the expected return, as indicated by the mean of the distribution of ex post (realized) IRRs, is less than the pro-forma IRR. The realistic expected IRR is nearly 50

[17] Bokhari & Geltner (2016) find that most depreciation results from NOI effects rather than increase in the cap rate.
[18] Our simulations are run with 10,000 trials. This runs in only a few seconds in Excel on standard laptops.
[19] The initial MV estimate of $1000 is the PV of the pro-forma cash flows, which are assumed to be the means of the future cash flow probability distributions. By Jensen's Inequality, for linear functions, the mean of the function equals the function of the mean (of the arguments). The valuation distribution has a slight positive skew reflecting the convexity of the resale price as a function of the symmetric distribution of future cap rates (going-out yields). But the centering of the probability around the pro-forma cash flows preserves the mean of the ex post PV outcomes as equal to the ex ante PV of the mean cash flow expectations.

basis-points below the pro-forma IRR of 7 percent. The ex post IRR distribution is symmetric around an expected total return of approximately 6.6% per annum, with a standard deviation of approximately +/- 300 basis-points. This reflects the fact that, for a given up-front investment price, the IRR is a concave function of the cash flows.[20]

The results described above and shown in Exhibit 5 do not reflect the flexibility the investor has to choose when to sell the property. Suppose we relax our assumption of the inflexible 10-year holding of the investment. Suppose we model a simple resale decision rule. In particular, we consider a "stop-gain" rule that sells when, and only when, the ex post realized pricing results exceed the original pro-forma projection (in any given year) by 20%. The result is reflected in the blue distributions in Exhibit 6, which are presented side-by-side on the same scale with the corresponding distributions without flexibility (in orange, as described above).

If we recognize resale timing flexibility in this manner, under the price dynamics assumptions described in Section 2, the mean ex post present value of the investment is no longer the $1000 market value, but rather $1250, that is, 25 percent greater than the market value, applying the 7 percent OCC as the discount rate. In only about 20 percent of the trials does the inflexible 10-year hold beat the flexible resale timing, ex post, under our simple myopic decision rule.[21] The effect on the simulated realized ex post IRR distribution is also very dramatic, increasing the ex post mean by almost 800 basis-points to approximately 14.5 percent per annum, based on the $1000 market value price. The ex post IRR distribution displays a strongly positive skew. The median ex post IRR is a little over 10 percent (which is still over 300

[20] Most real estate investors and investment analysts probably do not realize this technical point. By Jensen's Inequality, the greater the uncertainty surrounding the future cash flows, the more the unbiased probabilistic expected IRR will be below the projected IRR in an unbiased cash flow pro-forma for the investment.

[21] And it seems likely that a more thoughtful flexible resale decision rule than our automatic 20% stop-gain could improve on many of those 20% of the outcomes.

basis-points greater than the pro-forma IRR at the $1000 price). The positive skew in the IRR indicates the flexibility to profit from upside opportunities, at least in terms of the per annum IRR. However, even the downside outcomes, the left-hand tail of the ex post performance distributions (or "Value at Risk" – VaR), is improved by resale timing flexibility, right-shifted in the graphs. The fifth percentile of the valuation distribution (5% VaR) improves from less than $700 to around $850, and the 5th percentile ex post IRR adds some 400 basis-point, from less than 2% with no flexibility to almost 6% considering the flexibility. Of course, the ex post IRR distribution with the flexible resale timing reflects various different holding periods. Sometimes the property is sold after only two or three years, while occasionally it is held 20 years or more. The extremely positive per annum IRR results in the right-hand tail of the distribution correspond generally to very short, opportunistic holding periods of two, three, or four years. However, the mean holding period with the 20% stop-gain resale decision rule is about 10 years, very similar to the traditional pro-forma assumption.

Exhibit 7 shows the results of a sensitivity analysis of the above-described investment performance effects, varying many of the price dynamics parameter values as described in Section 2. The above described investment results are generally pretty robust to the price dynamics assumptions. Only if the cycle phase is initially in the mid-cycle headed down does the average valuation result fall below that of the inflexible 10-year hold. (See the left tail of the orange line in the top panel of Exhibit 7.) Even then, the expected IRR result is no worse than the inflexible 10-year hold. Interestingly, the two sensitivity analyses in which our stop-gain resale decision rule has virtually no effect on the valuation of the investment are the only two analyses in which we replace our real estate price dynamics with typical stock market price dynamics (Analyses 9 & 10, labeled "StkMktVol" and "StkMktTrig"). In those analyses the price dynamics follow a nearly pure Random Walk, with annual volatility varied between 10% and 50%, and the stop-gain trigger value varied from -20% to +40%. What drives the ability of the stop-gain decision rule to add value are the particular aspects of the price dynamics that are specific to private real estate as distinct from the stock market, such as the cyclicality and mean-reversion.

How should we understand the fact that a very simple (even simplistic) type of resale timing decision rule, reflecting a basic type of flexibility that seemingly any investor could have, adds 25 percent to the implied present value (comparing the simulated ex post mean PV to the estimated MV in the pro-forma)? Why is the market value not more like the $1250 expected ex post valuation? Why aren't more investors willing to pay $1250, and fewer property owners of the "Plain Vanilla Buildings" of the world willing to sell for less than $1250? Or is the realistic OCC greater than 7 percent? But the numbers in the pro-forma in Exhibit 4 are typical and realistic for "plain vanilla" stabilized investment properties in the US in the mid-2010s. What are we missing?...

The answer would seem to be that our flexibility-based $1250 valuation (25% above the market price) reflects *private valuation*, as distinct from market value. Let us refer to it as the "Investment Value" (IV) relevant in some sense for specific investors, or a specific type of investor, in contrast to "market value" (MV) that reflects the equilibrium price prevailing in the asset market. The IV is relevant for an investor who can realistically implement the 20% stop-gain resale rule, and who is not concerned about the possibly very short, or long, holding period implied by such discipline. This would not be all investors. And even though the 20% stop-gain rule may be no more unrealistic than the pro-forma assumption of the 10-year hold, the market equilibrium asset price could very well reflect various other concerns of the marginal investors who determine the equilibrium price in the market. A buyer who has the flexibility and information and discipline to effectively implement the 20% stop-gain rule, or some other (perhaps even better) decision rule, may indeed reap some private valuation surplus (positive NPV from the IV perspective) by buying at the market value. This does not imply that the MV is "wrong" in the context of market equilibrium valuation. It does not necessarily imply that the seller is experiencing a negative NPV by selling for $1000.

4. The Value of Flexibility in Multi-Asset Development Projects

Let us now turn to the analysis of an archetypical large-scale multi-asset development project. Development is the most creative type of real estate investment, and the type that has most impact on the physical built environment. Development also typically affords more types of flexibility than stabilized income property investment. And development projects are more rare and more unique than stabilized properties. This makes the estimation, even the conceptualization, of something like an equilibrium "market value" more difficult to define or evaluate for a development project. We begin by posing a typology of types of flexibility, or "real options", that typically may exist in large-scale projects producing multiple assets over time. Then we will introduce an archetypical such project that we will use to explore the value of the various types of real options, separately and together.

4.1 A Typology of Development Project Flexibility

At least five major types of flexibility, or real options (we use the terms essentially interchangeably in this context), can often exist in multi-asset projects.

1. Start Delay. This refers to the developer's (landowner's) right, without obligation, to delay the commencement of construction of the overall project. This option is very widely available in practice. It can be viewed as one aspect of a broader option, which is the general "call option" nature of land ownership, which provides the landowner with the right without obligation to start a development project whenever s/he chooses, based on the as-of-right zoning of the site. This is the type of option that has been extensively studied in the urban economics literature, and which may underlie a substantial portion of the value of vacant, developable urban land.[22] In practice, much preliminary work must be done by the developer before a large, complex project can possibly be started. Typically this requires assembly and permitting of the site, as well as design and specification of the project. By the time this preliminary work is complete, the call option would typically be viewed by the developer as a delay option, as there would normally have been a target start date in mind during the preliminary work. In any case, this option refers just to the commencement of the project construction, which may often begin with construction of infrastructure that is necessary for all or much of the subsequent buildout.

2. Modular Production Delay. This refers to the ability to pause the buildout of the project *after* construction has begun. The "modular" nature of this option is meant to suggest a substantial degree of flexibility to stop the project at any point, and then subsequently restart it again at that point. For example, a tract housing development may produce buildable lots in a modular manner that allows such flexibility. Some multiple-building high-rise projects, such as those typifying many emerging market cities, may also have this type of flexibility.

3. Phasing. This is another type of production timing flexibility applying to the buildout of the project after construction has started. Phasing is similar to modular production delay, but with less flexibility, because the nature of a "phase" is that it must be completed once construction on it has started. In other words, the project may be divided into sequential phases,

[22] As noted, see the strand of literature beginning with Titman (1985).

and there is flexibility to delay the commencement of any phase after the first, but any phase once started must be completed. Generally even very large projects will have at most only a few discrete phases. Each phase must be sufficiently self-contained so that it could stand on its own, given the prior and prerequisite buildout that must be completed (or committed to).

4. Product Mix Flexibility (Switching Option). This is a different type of option, what may be referred to as a "product option" as distinct from the "timing options" described above. The timing options do not envision flexibility in the physical product to be produced, only in the timing of its production. Product mix flexibility refers to the ability of the developer to change the type or design of structure up to just before commencement of its construction (or in some cases even after beginning construction). For example, housing and hospitality might be substitutes, or office and warehouse, or rental and condominium. At a smaller scale, even just changing the mix of types of units in a apartment or housing development, or in a hotel or office, provides product flexibility. In the pure sense, the product mix flexibility does not include timing flexibility, but only product type flexibility. In reality, there would typically also exist at least some timing flexibility, at a minimum, time enough to implement the product switch option decision. But for analytical purposes we can ignore timing flexibility and analyze pure product switching flexibility, by modeling more than one property asset sub-market with less than perfect correlation between their simulated price dynamics.

5. Horizontal or Vertical Expansion Options. This final type of flexibility is also a type of "product option" as distinct from the "timing options" considered in the first three types of flexibility listed above. Horizontal expansion is possible if the developer has control over the necessary extra land in or around the project "Base Plan". The "Base Plan" refers to the part of the project that the developer is very strongly committed to building (albeit possibly with the types of timing and product flexibilities noted above). The expansion options represent additional product buildout beyond the Base Plan, including components that the developer is not initially nearly as committed to constructing. Apart from this difference, expansion options may be conceived of and analyzed much like subsequent sequential phases (as described in (3) above). Vertical expansion options do not require additional land, but they require building design and construction and programming such that additional floors can be added atop an already completed and operating lower structure.[23]

4.2 An Archetypical Multi-Asset Development Project for Simulation Analysis

As we did in our analysis of the value of resale timing flexibility in a stabilized income property, we will now introduce an illustrative example multi-asset development project that we can use to explore the value of some of the types of flexibility described above. Of course, development projects are more unique than typical stabilized assets, so it is a bit more difficult to define a "plain vanilla" example. But the project depicted in Exhibit 8 is typical enough to serve as a useful foil. It is in fact a simplified and stylized representation of an actual project. We will dub this hypothetical version of the project, "Vanilla Terrace". Exhibit 8 shows the pro-forma cash flow projection in a manner typical of current industry practice. The lighter-shaded rows also present the physical quantities of production in the Base Plan temporal profile, and the bottom of the Exhibit is a bar-chart representing the this planned temporal pattern of physical production, of infrastructure and the product buildout.

Vanilla Terrace is a condominium development project involving numerous individual buildings. For simplicity, we assume that the condos can be built in either of two different

[23] Vertical expansion options are rare, but by no means unheard of. A famous example is the Health Care Services Corporations headquarters building in downtown Chicago, which expanded from 30 to 57 stories over 10 years after its first phase was built and fully operational.

configurations, labeled Type “A” and “B”, which would appeal to two different segments of the housing market. The initial Base Plan envisions an equal share of both types of units at all stages of production.

Vanilla Terrace is projected to produce and sell 850 housing units over a nine-year construction period. The intensity and rate of production peaks in the early-mid years, and then is planned to taper off, with very little in the last two years. Infrastructure must be built first. The time frame and temporal profile of Vanilla Terrace is typical of such large-scale, multi-asset projects. We assume that the condos can be pre-sold, and deposits received, one-third of the price in advance. Subsequent payments are due in the immediately following two years, one-third the year construction starts, and the last third when the 2-year construction process is completed on each condo. This allows the net cash flow projection to be at least slightly positive every year starting in Year 1 (see line 19 in the Exhibit table).[24] In the pro-forma, the only year with negative cash flow is dubbed “Year 0” (the “present”), which is when the land is purchased. The purchase price for the land is $200 million. The base case pro-forma reflects a projected growth rate in both housing prices and construction costs of 2 percent per year (the same as we applied to our stabilized asset in Section 3). As in Section 3, we treat the pro-forma cash flow projection as realistic and unbiased, assumed to approximately reflect the means of the probability distributions of the future cash flows.

Based on the Exhibit 8 pro-forma, the present value of the Vanilla Terrace housing units to be built, the product “buildout” of the project, is $882.6 million, discounting the projected revenues at a 7 percent OCC (the same rate as we applied to our stabilized income property

[24] An alternative financing technique in which the developer uses construction loans to finance the early stages of construction would not substantially affect the economic analysis, as lending is generally a zero-NPV exercise.

investment in the previous section). The present value of the projected construction costs for both infrastructure and housing is $657.9 million, discounting at a lower rate of 3 percent appropriate for the OCC of construction costs.[25] This gives the project a Net Present Value (NPV) of $224.7 million, exclusive of land cost. In principle, this is the market opportunity value of the land, assuming that Vanilla Terrace is the Highest and Best Use (HBU) of the land, and that it is "ripe" for development (that it does not make sense to continuing with land speculation). Thus, the land value is approximately 25% of the total gross present value of the assets to be built, which is a typical land value fraction for development projects in the US in the mid-2010s. At this $224.7 million market value for the project (the maximum bid price for the land, giving NPV=0 net of land cost), the pro-forma IRR is 18.01% per annum, which may be taken as the OCC of the development project *per se*. At the given land price of $200 million, the all-in NPV (net of land cost) is positive $24.7 million, and the pro-forma IRR is 22.6%. The numerical values and project economics of Vanilla Terrace are not untypical of real estate development projects in the US in the mid-2010s.

In order to use Vanilla Terrace as a basis to explore the value of flexibility in such a development project, we will assume that any one or combination of three possible types of flexibility may exist for the project: Start delay, Modular production delay, and Product mix

---

[25] The OCC rate appropriate for translating construction cost amounts across time is generally only a point or so above the riskfree rate. This is because construction costs (prices) lack much volatility and correlation with real estate markets or other financial markets. (In CAPM terms, the construction cost "beta" is very low.) See our discussion in Section 2, as well as Gyourko & Saiz (2006), Wheaton & Simonton (2007), and Chapter 29 in Geltner *et al* (2014). We note that the OCC of construction costs is not the same thing as the OCC faced by the lender of a construction loan, and furthermore that the interest rate on a non-recourse loan may be considerably higher than the realistic expected return on the loan. Note further that by discounting the expected construction costs at a low rate, we increase the present value of this cost item, which is a negative component of the project NPV, hence, we certainly do not minimize the "risk" posed by "getting the construction costs wrong." This conception of "risk", however, is related to expectations, and is different from the capital market conception of risk as it relates to determining the equilibrium cost of capital, which is the OCC rate appropriate for translating money amounts across time.

switching flexibility. This will allow us to examine each of these three types of real options separately, as well as together. Of course, our analysis will apply directly only to this one specific illustrative example project. But at a broad-brush level, Vanilla Terrace has many typical features of large, multi-asset development projects, and its project economics and pro-forma investment performance projections are typical of such projects in the US in the mid-2010s.

4.3 The Value of Timing and Product Flexibility in the Vanilla Terrace Project

In this section we report the results of Monte Carlo simulation analysis of the three types of flexibility noted above. The analysis uses 10,000 independent randomly generated trials of 24-year future histories, allowing substantial possible delay in either (or both) the start of the project and its modular buildout. The analysis reflects the real estate price dynamics described in Section 2, and we also report the results of sensitivity analysis on many of the specific price dynamics parameter assumptions described in that Section. The underlying probability distributions are calibrated to cause the simulated mean ex post NPV of the project without any flexibility (that is, following the Base Plan of Exhibit 8 exactly in timing and product) to closely approximate the pro-forma NPV of roughly +$25 million at the $200 million land price. This is to reflect the assumption that the pro-forma is a "good" (state-of-the-art) cash flow projection and DCF valuation, reflecting the means of the future cash flow probability distributions and the actual OCC discount rates.

A key metric of investment performance in our simulation analysis is the NPV as of Time 0 of the simulated ex post cash flow outcome of the project. We study the sample distributions of this output metric, along with those of the realized IRR for the project (assuming the given land price of $200). We also examine the distribution of the amount of completion delay in the project, relative to the Base Plan's nine-year schedule. In computing the ex post NPV of the project, we apply as the discount rate an exogenous "hurdle rate" of 18 percent. This rate is in a rigorous economic sense ad hoc, because it is applied to an ex post result. However, the 18 percent rate closely approximates the OCC derived from the economic analysis of the project as described above (which is 18.01%).

To model the decision flexibility relating to the three types of timing and product decisions noted above, we employ simple, myopic decision rules similar in spirit to the stop-gain rule introduced in Section 3. In general, we use "IF()" statements in Excel® to compare the simulated ex post profit outcome with the pro-forma projection (based on simulation of both housing prices and construction costs). The project implementation is modified accordingly in each year. That is, the start of the project may be delayed, or its continued production may be paused, or its mix of Type "A" versus "B" housing units may be varied. Price dynamics are simulated independently for each type of housing, but with strong positive correlation (+80%) between the two sub-markets.[26] We make no claim that the simple decision rules we have programmed in Excel for this analysis are optimal, or even very good. As noted, we describe this type of modeling as "behavioral analog". We believe the model approximates the type of flexible decision making that developers often could, and perhaps do, implement in practice.

Exhibits 9-12 show the results of typical 10,000-trial Monte Carlo frequency distributions of the investment performance indicators: ex post NPV and IRR. The orange curves represent the Base Plan without flexibility. The blue curves represent the results with decision flexibility. Exhibits 9-12 depict, respectively: Start delay flexibility only, Modular buildout delay flexibility only[27], Product switching flexibility only, and finally (in Exhibit 12) all three types of flexibility

[26] Other things equal, the greater this correlation, the less valuable is the product switching option.
[27] That is, delay possible only *after* the project has been started with substantial infrastructure construction committed.

co-existing together. The overall result in terms of the central tendencies is summarized in the table in Exhibit 13, which includes all seven possible combinations of the three options, and also shows the average project completion delays.

Examining the investment performance outcome frequency distributions, a few general comments are in order. First consider the pure effect of uncertainty, without flexibility (the orange curves). The general shape and central tendency of the effect of uncertainty is as described in Section 3 for the stabilized asset. The valuation distribution (NPV) has a slight positive skew, but its mean is identical to the pro-forma valuation (NPV = $25 million).[28] The IRR distribution is largely symmetrical but with a mean below that of the pro-forma IRR.

Next examine the comparison of the blue curves and the orange curves, showing the effect of flexibility. The blue curves are all right-shifted. The mean outcome (expected investment performance) is substantially improved by the types of flexibility and simple myopic decision-making modeled in the simulation. In the case of the two timing options (start delay and subsequent buildout delay), the right-shift occurs only in the left-hand tail. The right-hand tail of the distributions, corresponding to upside outcomes, are effectively the same with or without flexibility. In other words, when history turns out very well, the Base Plan schedule is carried out without any extra delay or pauses. Project delay flexibility, whether in starting the project to begin with or in carrying through its completion, is a “defensive” type option. It acts a bit like insurance. It helps to protect against the worst downside outcomes by enabling the developer to reduce expenditures during periods when selling the condos would not be profitable (or would be less profitable than the pro-forma projection). The product switching option is a bit different. It

[28] This reflects our assumption that the pro-forma is unbiased. Minor variation in the outcome sample mean around the pro-forma NPV simply reflects random sample outcomes and are not statistically or economically significant.

provides both downside protection and upside opportunities. The switching option is like a combination of both a put and a call option: a put on the product type being switched away from, and a call on the product type being switched to. This is therefore both a “defensive” and “offensive” option. We see in Exhibit 11 that the blue curve depicting the switching option is right-shifted all along the outcome range compared to the orange curve, not just in the left-hand tail.

The magnitude of the effects of flexibility is summarized in the central tendencies reported in the table in Exhibit 13. The table shows only the differences in the outcome sample means between the flexible and inflexible cases. Flexibility adds considerably to the value of the project. As a percentage of the economic value of the project (the land bid-price of $225 million), timing flexibility adds between 13% and 21% to the value. Start delay flexibility alone adds 21%. Including buildout delay flexibility as well as start delay flexibility does not add any additional value. The two types of timing flexibility are redundant as far as the central tendency of the NPV is concerned. (Including buildout as well as start delay flexibility does improve the IRR outcome slightly, especially on the downside tail.) But what if the project does not have start delay flexibility? Then buildout delay flexibility adds 13% to the project value. It does not add as much value as the overall project start delay flexibility because buildout delay does not avoid much of the infrastructure construction cost, which must be built at the outset. The switch option by itself (with no delay flexibility) adds slightly more value, 13.6%. (Of course, this would be very dependent on the type of product type flexibility that exists, and how correlated the markets for the different products are.) While the two types of timing flexibility, start delay and buildout delay, are essentially redundant, the product type flexibility is additive to the timing options. The switch option and the buildout delay option together add 21% to the project value. The switch option and the start delay option together (without buildout delay) add 34% to the project value, the same as what is added by all three options in combination.

In general, the IRR improvement resulting from flexibility is more impressive than the NPV improvement. (See the middle column in Exhibit 13.) The combination of start delay flexibility and the switching option adds 1200 basis-points to the expected return. Furthermore, the downside protection in the IRR from the delay options is particularly strong. (Note the striking right-shift in the left-hand tail of the blue distributions In Exhibits 9, 10 & 12.)

Project completion delay is also an important consideration. The right column in the Exhibit 13 table shows considerable variation in the effect on the completion of the (originally planned) nine-year project. The average start delay if that option is available is less than two years (only 1.5 years if buildout delay is also available). Of course, the switching option by itself causes no delay. But the buildout delay option can extend the project quite long, an average of five years by itself or six years if combined with the start delay option. All three options together produce an average completion delay of almost five years. However, as noted, there is no value added by the buildout delay option if the start delay option is present. In that case, one gets as much value from the combination of just the start delay and switching options, and that combination produces an average completion delay of only 1.5 years.

Finally, Exhibit 14 depicts the results of a sensitivity analysis of the price dynamics assumptions underlying the simulations for Vanilla Terrace. The ranges of parameter assumptions reflected in the sensitivity analysis were noted in Section 2.[29] The general implication of the results shown in Exhibit 14 would seem to be that, while the specific

[29] Note that Analyses 7 and 8 test different decision rules rather than different price dynamics, and Analyses 9 and 10 test “stock market price dynamics” (essentially, a pure Random Walk).

performance implications of development project flexibility are clearly sensitive to the nature of the underlying price dynamics, in general the conclusion that flexibility improves investment performance by a substantial amount is pretty robust, particularly in terms of the expected return (IRR). The major exception is if the price cycle is in a particular phase: mid-cycle headed up. (This corresponds in Exhibit 14 to the left end of the orange line, Analysis 6.) In that case, with our simple myopic decision rules, the delay options harm the results (though the switching option is benign). But in reality, if it were obvious that the market was in the midst of an upswing, developers would likely not delay the project (unless forced to by lending policies by construction lenders or by political/regulatory hurdles).

5. Conclusion

This paper has posited a thoughtful model of real estate price dynamics, and applied that model to explore the value of flexibility in a variety of real estate investment contexts. Our approach differs from traditional real options analysis based on formal economic equilibrium pricing theory. Our "behavioral analog" approach enables a much broader and more practical analysis of the effect of flexibility on investment performance in both stabilized properties and a common type of development project. It also allows a simpler, more transparent analysis process, using common spreadsheet software, in a manner that should make the analysis more accessible to non-technical decision makers.

We find that flexibility is an important component of the value of real estate investments and development projects. Resale timing flexibility could add 25% to the value of a typical property, though this is from a private valuation perspective. In development projects, once the developer is ready to go with the project, the type of timing flexibility that is most relevant is probably delay flexibility, either in the start or the buildout of the project. We find that delay options are essentially redundant with each other, but additive with product type switching options. Such options in combination add over one-third to the net value of the development project (the economic value of the land), and the delay options provide significant downside protection. Expected returns, in the form of the project's IRR, are bolstered by several hundred to over a thousand basis-points by the effect of flexibility. Subsequent buildout delay flexibility, after the start of the project, can result in significant completion delays of up to several years on average in our simulations.

Arguably, the type of flexibility based components of investment value and return expectations explored in this paper are widely ignored in contemporary analytical practice in the real estate investment industry as well as in appraisal practice. The type of simulation used in this study is almost never applied in practice, nor is the nature of real estate price dynamics widely appreciated by industry analysts.

In closing, we should raise one last point about our analysis. We have modeled very simple (and even simplistic) decision rules in our behavioral analog approach. Essentially we apply myopic decision rules based on the comparison of current prices to those in the initial pro-forma projection. Quite possibly developers in the real world exercise better decision making about the flexibility that they have. We don't think that developers are ignorant about the existence and usefulness of flexibility. (They just don't analyze it explicitly or quantitatively as we have in this paper.) If developers are better decision makers than our automatic simple myopic rules, then our results may understate the value of flexibility that exists in real projects. Furthermore, there may exist other types of flexibility besides the three that we have explicitly considered in Section 4. On the other hand, we may be ignoring some costs and constraints related to the exercise of flexibility in development project production in the real world. In summary, at a minimum, the type of thoughtful and practical quantification of flexibility that we have explored in this paper deserves more use and attention in both the real estate academic and industry communities.

Bibliography

S Bokhari, D Geltner. "Characteristics of Depreciation in Commercial & Multifamily Property: An Investment Perspective." *Real Estate Economics,* 2016.

S Bokhari, D Geltner. "Commercial Buildings Capital Consumption and the United States National Accounts." *Review of Income & Wealth,* forthcoming, 2018.

D Capozza, R Helsley. "The Fundamentals of Land Prices and Urban Growth." *Journal of Urban Economics* 26: 295–306, 1989.

J Clapp, P Eichholtz, T Lindenthal. "Real Option Value over a Housing Market Cycle." *Regional Science & Urban Economics* 43: 862-874, 2013.

A Dixit, R Pindyck. *Investment Under Uncertainty*. Princeton University Press, 1994.

Eurostat. *Handbook on Commercial Property Price Indicators*. Eurostat, *forthcoming* 2017.

D Geltner. "Real Estate Price Indices and Price Dynamics: An Overview from an Investments Perspective." *Annual Review of Financial Economics* 7:615-633, 2015.

D Geltner, J Mei. "The Present Value Model with Time Varying Discount Rates: Implications for Commercial Property Valuation & Investment Decisions." *Journal of Real Estate Finance & Economics* 11: 119-135, 1995.

D Geltner, N Miller, J Clayton, P Eichholtz. "How Volatile Are Commercial Property Assets?", in Chapter 9 (p.188) of: *Commercial Property Analysis and Investments, 3e*. OnCourse Learning, 2014.

D Geltner, R de Neufville. *Flexibility and Real Estate Valuation under Uncertainty: A Practical Guide for Developers*. Wiley Blackwell, *forthcoming*.

R Grovenstein, J Kau, H Munneke. "Development Value: A Real Options Approach Using Empirical Data." *Journal of Real Estate Finance & Economics* 43: 321-335, 2011.

J Gyourko, A Saiz. "Construction Costs and the Supply of Housing Structure." *Journal of Regional Science* 46:661-680, 2006.

A Plazzi, W Torous, R Valkanov. "Expected Returns and Expected Growth in Rents of Commercial Real Estate." *Review of Financial Studies* 23: 3469-3519, 2010.

P Samuelson. "Proof that Properly Anticipated Prices Fluctuate Randomly." *Industrial Management Review* 6:41-49, 1965.

R Shiller. "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?" *American Economic Review* 71: 421-436, 1981.

S Titman. "Urban Land Prices Under Uncertainty." *American Economic Review* 75: 505–514, 1985.

L Trigeorgis. *Real Options: Managerial Flexibility and Strategy in Resource Allocation*. MIT Press, 1996.

W Wheaton. “The Cyclic Behavior of the National Office Market.” *Real Estate Economics* 15: 281-299, 1987.

W Wheaton. “Real Estate ‘Cycles’: Some Fundamentals.” *Real Estate Economics* 27: 209-230, 1999.

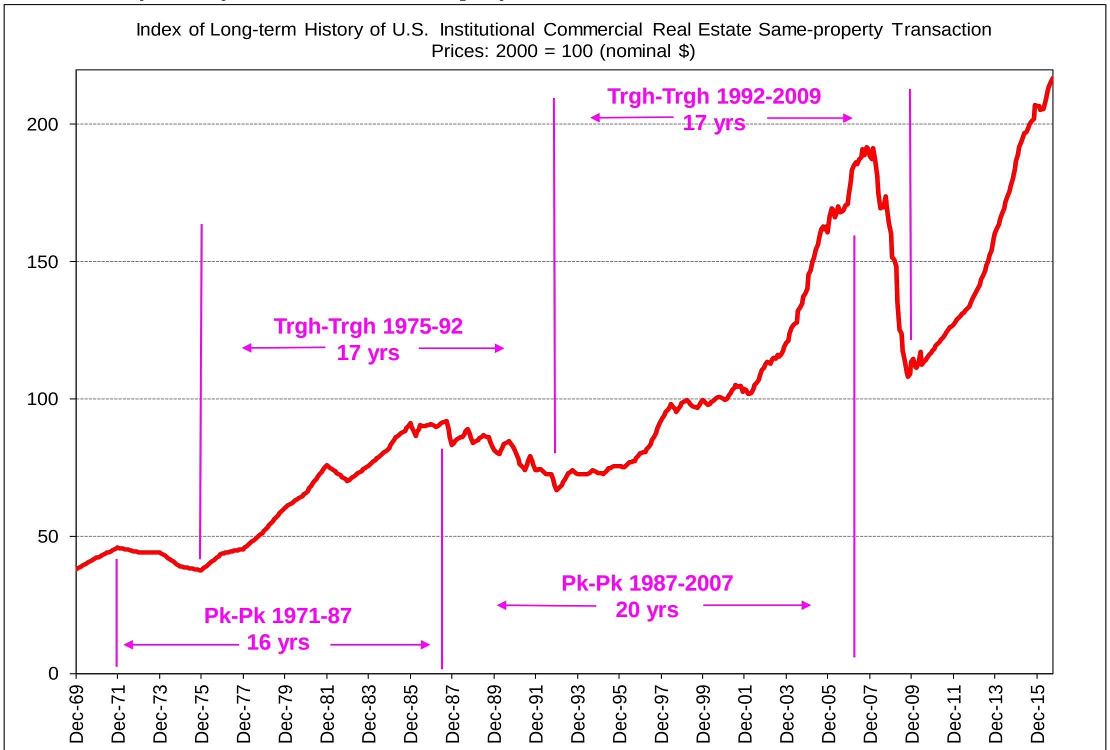
W Wheaton, W Simonton. “The Secular and Cyclic Behavior of ‘True’ Construction Costs.” *Journal of Real Estate Research* 29: 1-25, 2007.

J Williams. “Real Estate Development as an Option.” *Journal of Real Estate Finance & Economics* 4: 191–209, 1991.

P Yella. “Idiosyncratic Risk in US Commercial Real Estate.” Masters Thesis. *Massachusetts Institute of Technology*, 2016. (Available at https://dspace.mit.edu/handle/1721.1/106454.)

Exhibits

Exhibit 1: Cyclicality in US Investment Property Prices



*(Sources: Moody's/REAL, Moody's/RCA, TBI, Author's estimates.)

Exhibit 2: General Real Estate Price Dynamics Algorithm Used in Simulations

Long Run Trend Rent Level as of time "t":
$LRTRL_t = LRTRL_{t-1}(1+g)$
where:
g = Long-run trend growth rate (per annum).

No-Cycle De-trended Rent Return in Year "t":
$r_t = \varepsilon_t + \rho r_{t-1}$
where:
ε_t = Time "t" innovation, realization of volatility in period "t" (zero mean random variable);
ρ = Auto-regressive parameter (positive fraction).

No-Cycle Rent Level:
$NCRL_t = NCRL_{t-1}(1+g+r_t) + \lambda(LRTRL_{t-1} - NCRL_{t-1})$
where:
λ = Mean-reversion rate parameter.

Space Market Rent Cycle (at year "t"):
$C_t = 1 + (A/2)SIN((t-X)2\pi/P)$
where:
A = Amplitude (fraction of mean level);
P = Period (in years);
X = Phase (in years: 0 <= X <= P).

Rent Level as of time "t":
$RL_t = C_t NCRL_t$.

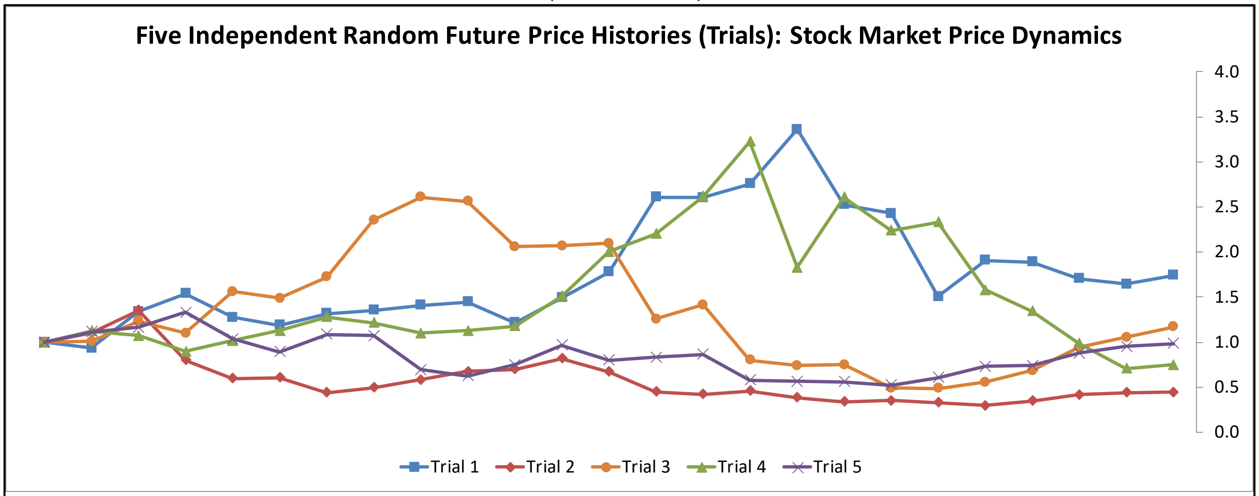
Yield in Asset Market as of time "t":
$y_t = Y - (a/2)SIN((t-x)2\pi/p)$
where:
a = Amplitude (in basis-points);
p = Period (in years);
x = Phase (in years: 0 <= x <= p).

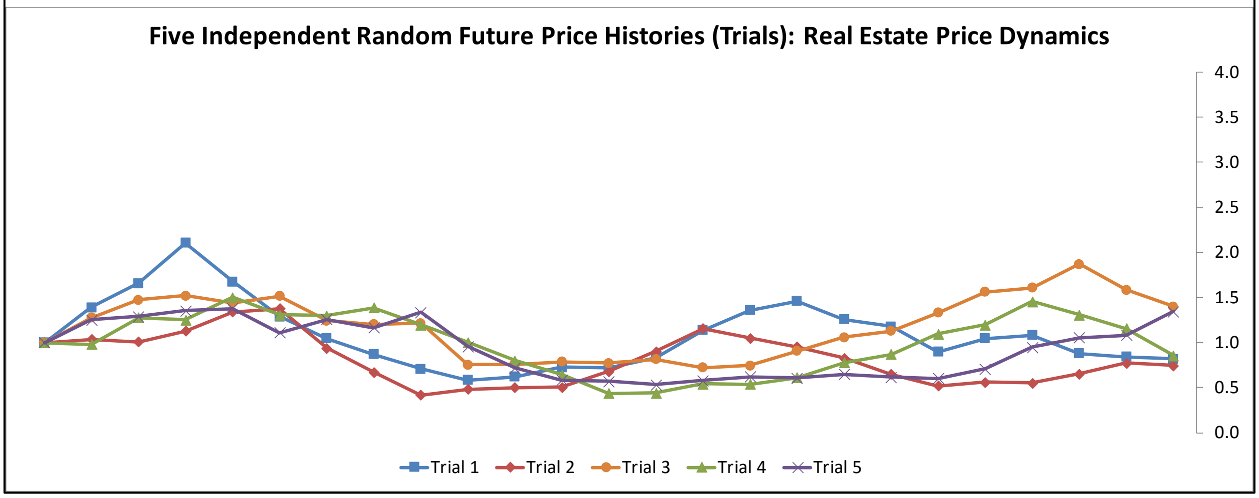
Asset True Value as of time "t":
$TV_t = RL_t / y_t$.

Asset Transaction Price as of time "t":
$V_t = TV_t(1+\eta)$
where:
η = Random "noise" (dispersion) realization (fraction of true value).

Note: In addition to the above components, V_t is subject to any realized "Black Swan" effect, which is a Bernoulli occurrence in any given year (no more than once per 24-year scenario history), with a specified fractional negative impact that decays geometrically at the mean-reversion rate, λ. Also note that the volatility realization, ε_t, includes both systematic (market) innovations as well as specific (idiosyncratic) innovations.

Exhibit 3:
Random Independent Trials:
24-year Annual Frequency Histories of De-trended Asset Price Dynamics*
(Same scale)



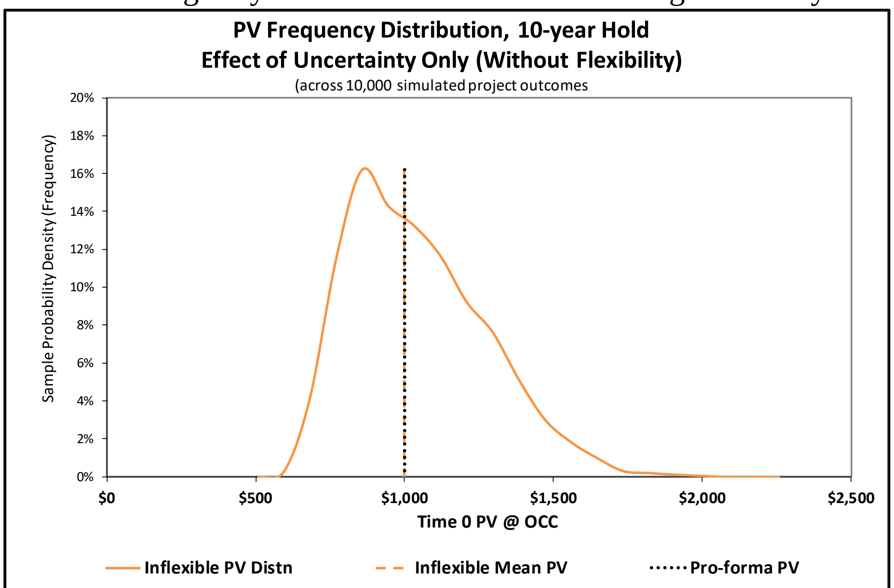


*Stock Market Dynamics based on Random Walk with 20% volatility. Real Estate Price Dynamics include all eight elements described in Section 2 with base case parameter values as described in Section 2, with initial cycle phase set to mid-cycle headed up. (Both sets include the Black Swan element as described in Section 2.)

Exhibit 4:
“Plain Vanilla Building” Cash Flow Projection Pro-forma & DCF Valuation

Plain Vanilla Building													
Expected Cash Flows:	**Totals:**	**Year 0**	**Year 1**	**Year 2**	**Year 3**	**Year 4**	**Year 5**	**Year 6**	**Year 7**	**Year 8**	**Year 9**	**Year 10**	**Year 11**
Projected Operations (Realistic Assumptions):													
Potential Gross Income (**PGI**)			100.00	102.00	104.04	106.12	108.24	110.41	112.62	114.87	117.17	119.51	121.90
Vacancy Allowance			5.00	5.10	5.20	5.31	5.41	5.52	5.63	5.74	5.86	5.98	6.09
Effective Gross Income **(EGI)**			95.00	96.90	98.84	100.81	102.83	104.89	106.99	109.13	111.31	113.53	115.80
Operating Expenses			35.00	35.70	36.41	37.14	37.89	38.64	39.42	40.20	41.01	41.83	42.66
Net Operating Income (**NOI**)			**60.00**	**61.20**	**62.42**	**63.67**	**64.95**	**66.24**	**67.57**	**68.92**	**70.30**	**71.71**	**73.14**
Capital Improvement Expenditures			10.00	10.20	10.40	10.61	10.82	11.04	11.26	11.49	11.72	11.95	12.19
Net Cash Flow from Operations **(PBTCF)**			**50.00**	**51.00**	**52.02**	**53.06**	**54.12**	**55.20**	**56.31**	**57.43**	**58.58**	**59.75**	**60.95**
PBTCF from Reversion												**1218.99**	
PBTCF Total (Including Reversion)			**50.00**	**51.00**	**52.02**	**53.06**	**54.12**	**55.20**	**56.31**	**57.43**	**58.58**	**1278.75**	
Time 0 PV @ OCC		**1000.00**											
Projected IRR @ Mkt Val Price	**7.00%**	-1000.00	50.00	51.00	52.02	53.06	54.12	55.20	56.31	57.43	58.58	1278.75	

Exhibit 5:
Simulated Ex Post Distributions of Plain Vanilla Building Present Value & IRR, Assuming 10-year Hold Without Resale Timing Flexibility



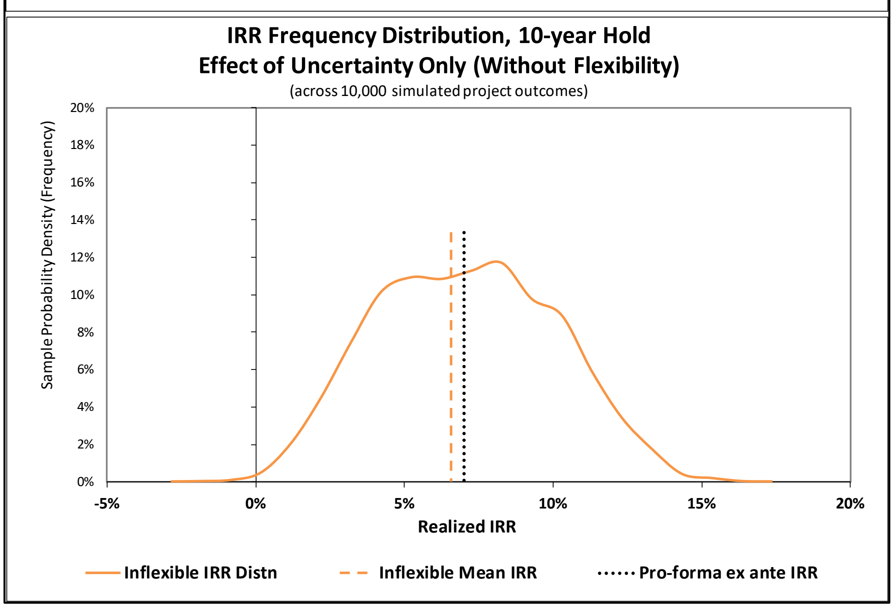
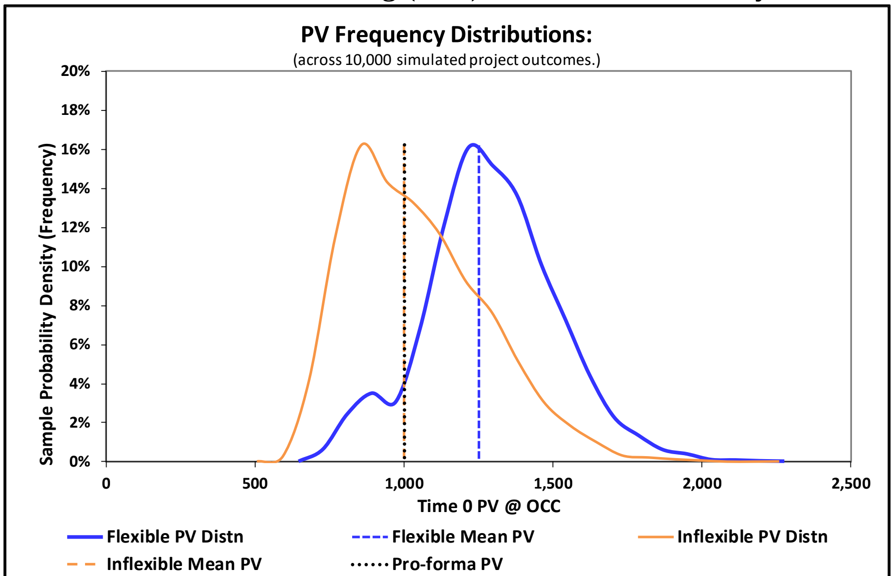


Exhibit 6:
Simulated Ex Post Distributions of Plain Vanilla Building Present Value & IRR,
Comparison of Flexible Resale Timing (blue) versus Inflexible 10-year Hold (orange)



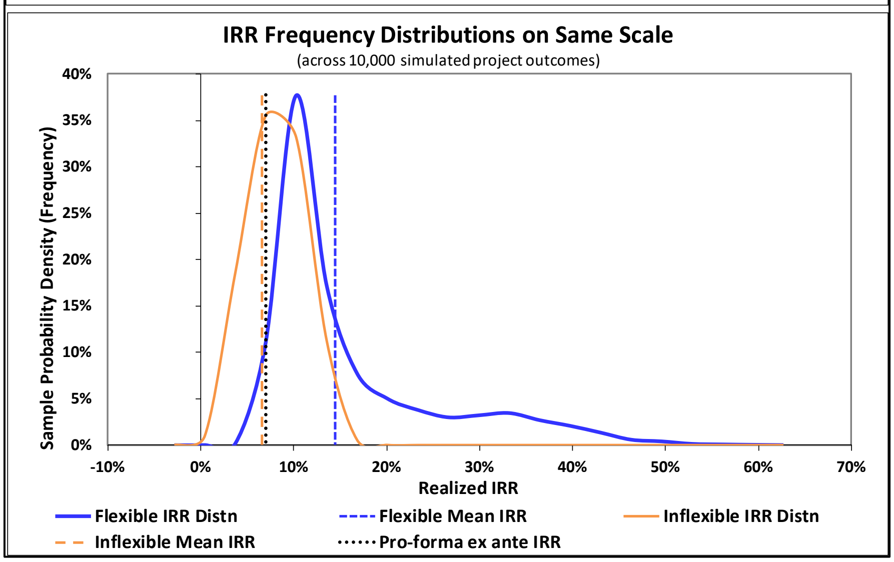
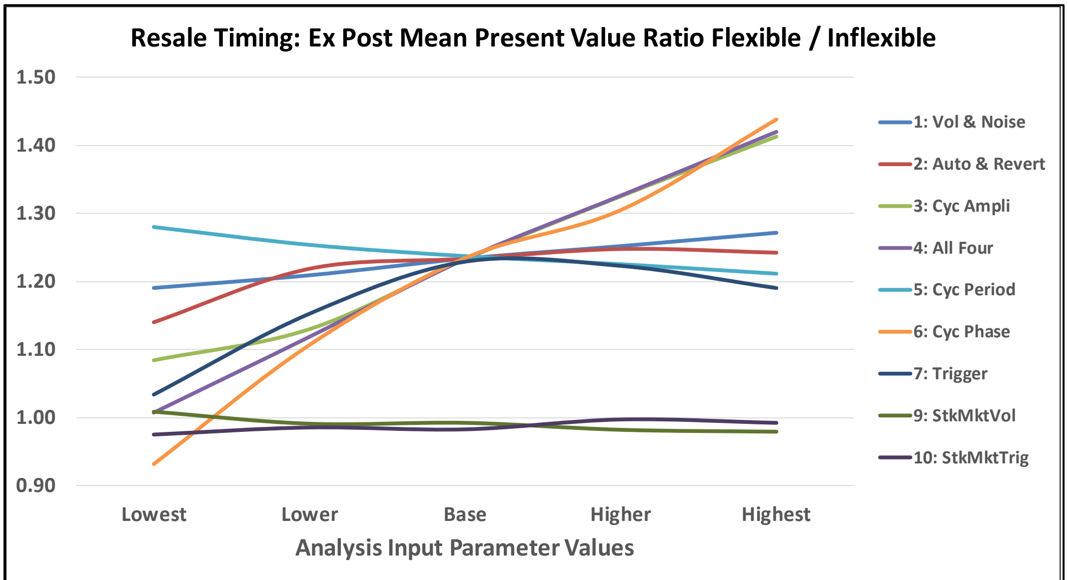


Exhibit 7:
Stabilized Asset Flexible Resale Timing Effect:
Sensitivity Analysis of Various Price Dynamics and Decision Rule Assumptions
on the Mean Valuation & IRR Outcomes



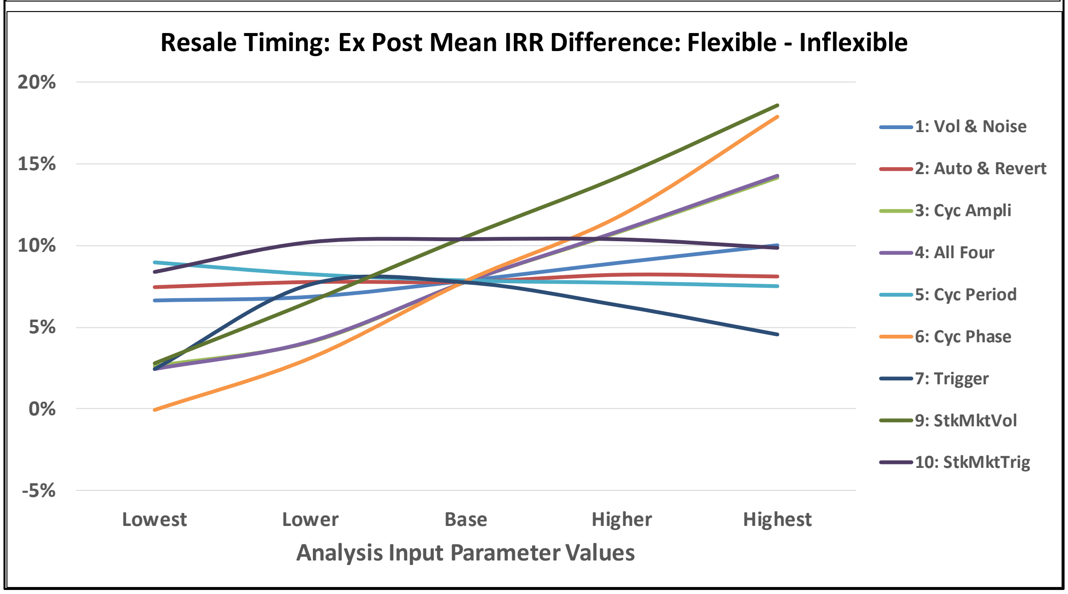


Exhibit 8:
"Vanilla Terrace" Cash Flow Projection Pro-forma & DCF Valuation

	Vanilla Terrace													
	DEVELOPMENT PROJECT PLAN	**UNITS**	**TOTALS**	**YEAR 0**	**YEAR 1**	**YEAR 2**	**YEAR 3**	**YEAR 4**	**YEAR 5**	**YEAR 6**	**YEAR 7**	**YEAR 8**	**YEAR 9**
1	**Total Sold**	Residences	850		144	392	138	96	62	10	8	0	0
2	Land Cost	USD '000s		$200,000									
3	Infrastructure Quantities	Residence Equivalent	240		80	80	80	0	0	0	0	0	0
4	Infrastructure Construction Costs	USD '000s	$156,080		$51,000	$52,020	$53,060	$0	$0	$0	$0	$0	$0
5	**Type A**												
6	Total Sold	Residences	425		72	196	69	48	31	5	4	0	0
7	Total Started	Residences	425		0	72	196	69	48	31	5	4	0
8	Total Completed	Residences	425			0	72	196	69	48	31	5	4
9	Type A Revenues	USD '000s	$558,738		$30,600	$115,566	$146,076	$137,124	$66,419	$38,256	$18,522	$4,261	$1,914
10	Type A Construction Costs	USD '000s	$287,806		$0	$23,409	$88,876	$89,639	$40,368	$27,802	$12,923	$3,295	$1,494
11	Type A Profits	USD '000s	$270,932		$30,600	$92,157	$57,200	$47,485	$26,051	$10,454	$5,599	$965	$421
12	**Type B**												
13	Total Sold	Residences	425		72	196	69	48	31	5	4	0	0
14	Total Started	Residences	425		0	72	196	69	48	31	5	4	0
15	Total Completed	Residences	425			0	72	196	69	48	31	5	4
16	Type B Revenues	USD '000s	$558,738		$30,600	$115,566	$146,076	$137,124	$66,419	$38,256	$18,522	$4,261	$1,914
17	Type B Construction Costs	USD '000s	$287,806		$0	$23,409	$88,876	$89,639	$40,368	$27,802	$12,923	$3,295	$1,494
18	Type B Profits	USD '000s	$270,932		$30,600	$92,157	$57,200	$47,485	$26,051	$10,454	$5,599	$965	$421
19	**Net Annual Cash Flows**	USD '000s	$385,784	-$200,000	$10,200	$132,294	$61,339	$94,971	$52,103	$20,907	$11,198	$1,931	$841
20	**NPV @ 18.0% Discount Rate**	USD '000s		$24,711									
21	**IRR**	USD '000s		22.57%									
22	**Project Economics**		**Time 0 PV:**										
23	Built Asset Revenues (7% OCC)		$882,588	=PV[V]	$61,200	$231,132	$292,151	$274,249	$132,839	$76,512	$37,043	$8,521	$3,829
24	Construction Costs (3% OCC)		$657,910	=PV[K]	$51,000	$98,838	$230,813	$179,278	$80,736	$55,604	$25,845	$6,591	$2,988
25	Implied Land Market Value		$224,678	=PV[V]-PV[K]		25.46%	=LVF						
26	**Implied Mkt OCC (as IRR)**		**18.01%**	-$224,678	$10,200	$132,294	$61,339	$94,971	$52,103	$20,907	$11,198	$1,931	$841

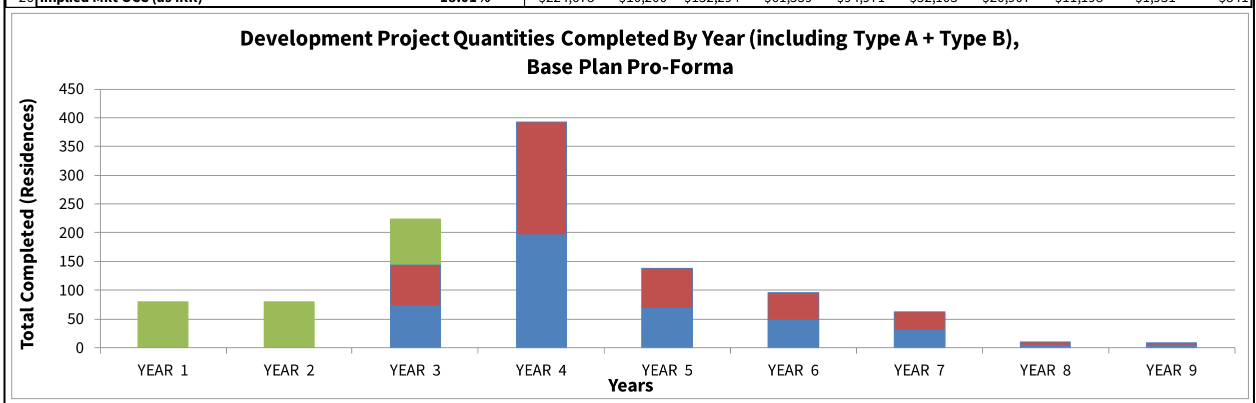
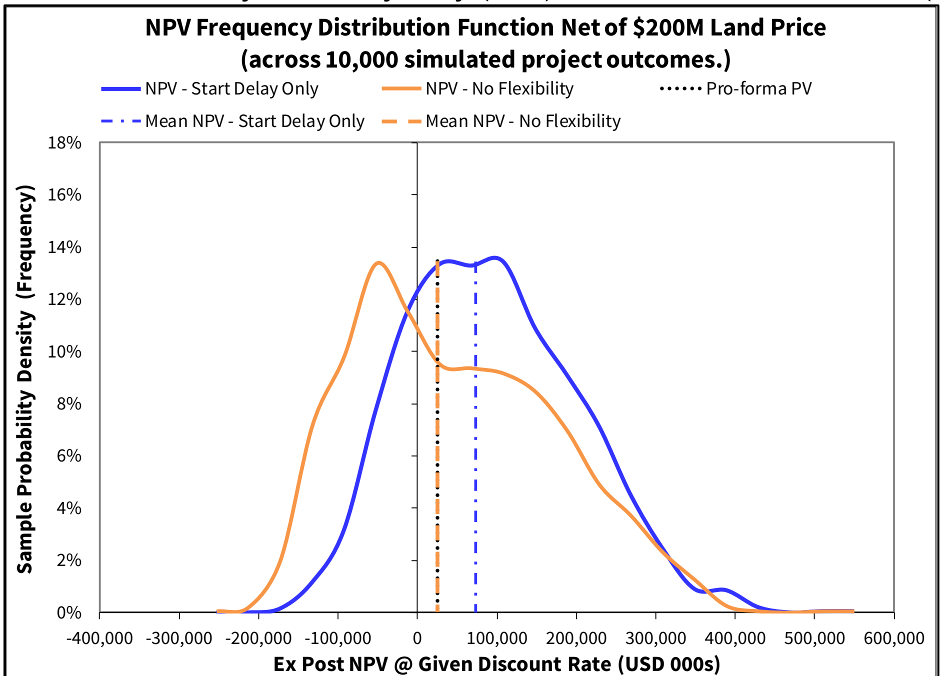


Exhibit 9:
Simulated Ex Post Distributions of Vanilla Terrace Development Project Present Value & IRR, Comparison of Start Delay Flexibility Only (blue) versus Inflexible Base Plan (orange)



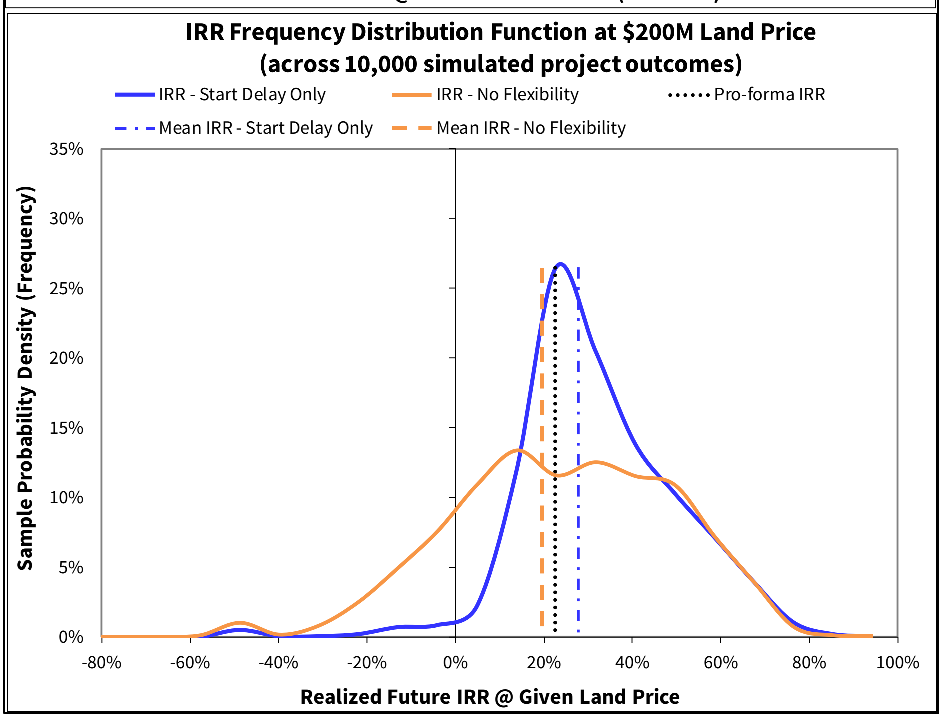
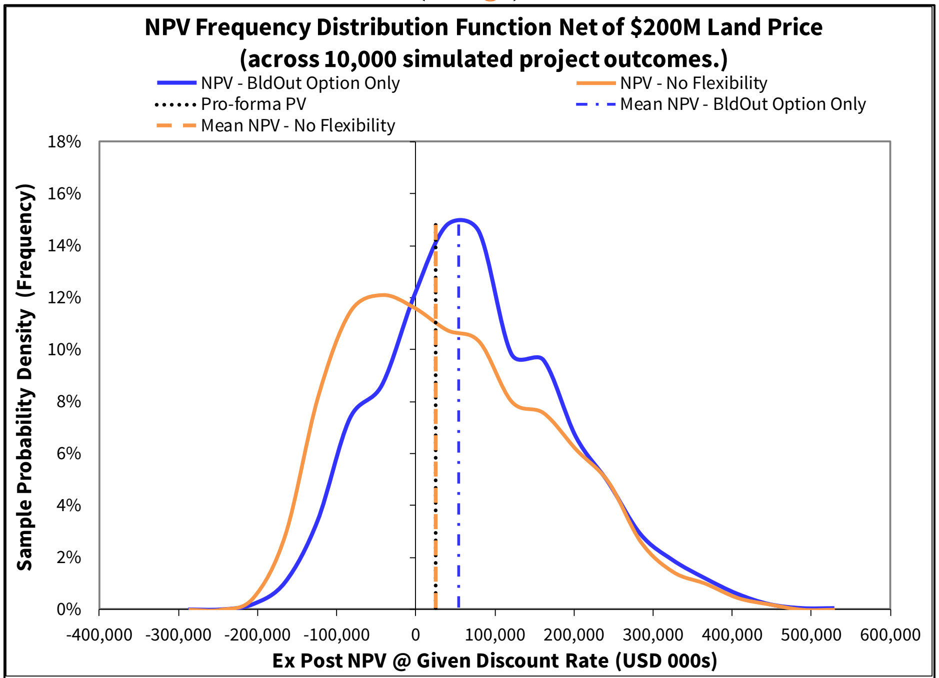


Exhibit 10:
Simulated Ex Post Distributions of Vanilla Terrace Development Project Present Value & IRR, Comparison of Modular Buildout Delay Flexibility Only (blue) versus Inflexible Base Plan (orange)



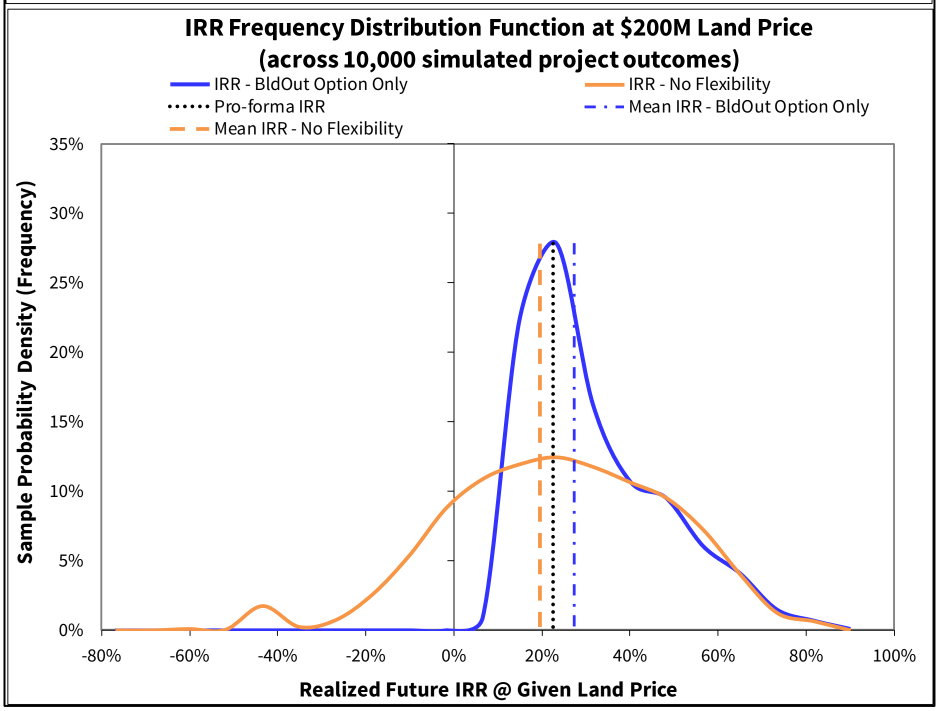
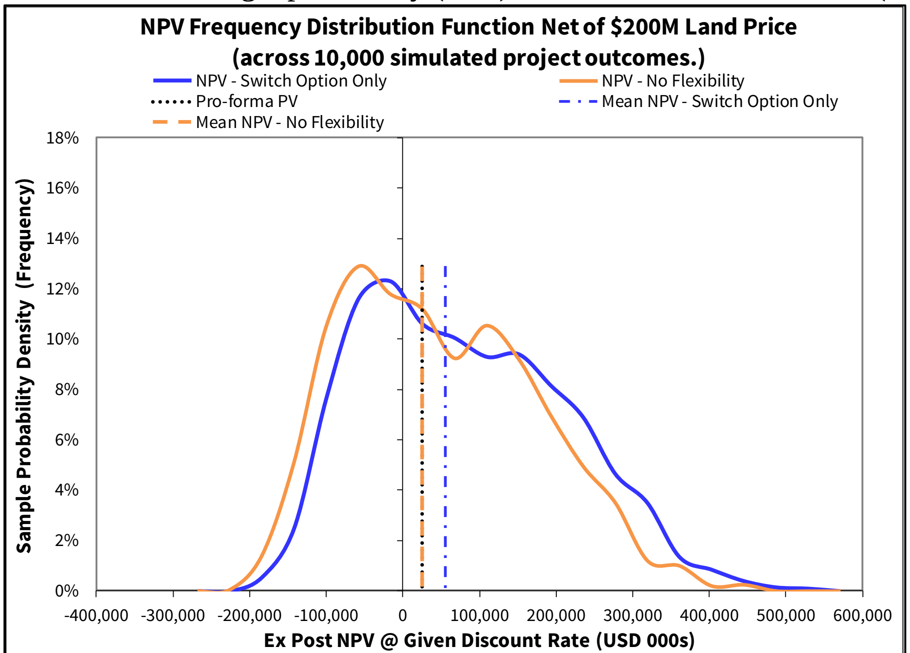


Exhibit 11:
Simulated Ex Post Distributions of Vanilla Terrace Development Project Present Value & IRR, Comparison of Switching Option Only (blue) versus Inflexible Base Plan (orange)



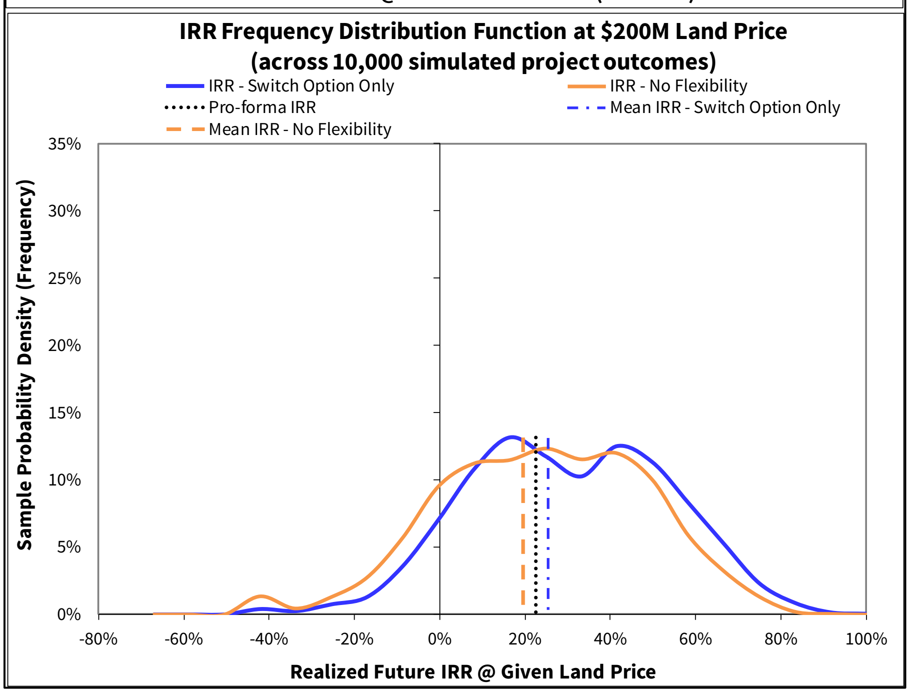


Exhibit 12:
Simulated Ex Post Distributions of Vanilla Terrace Development Project Present Value & IRR,
Comparison of All Three Options Together (blue) versus Inflexible Base Plan (orange)

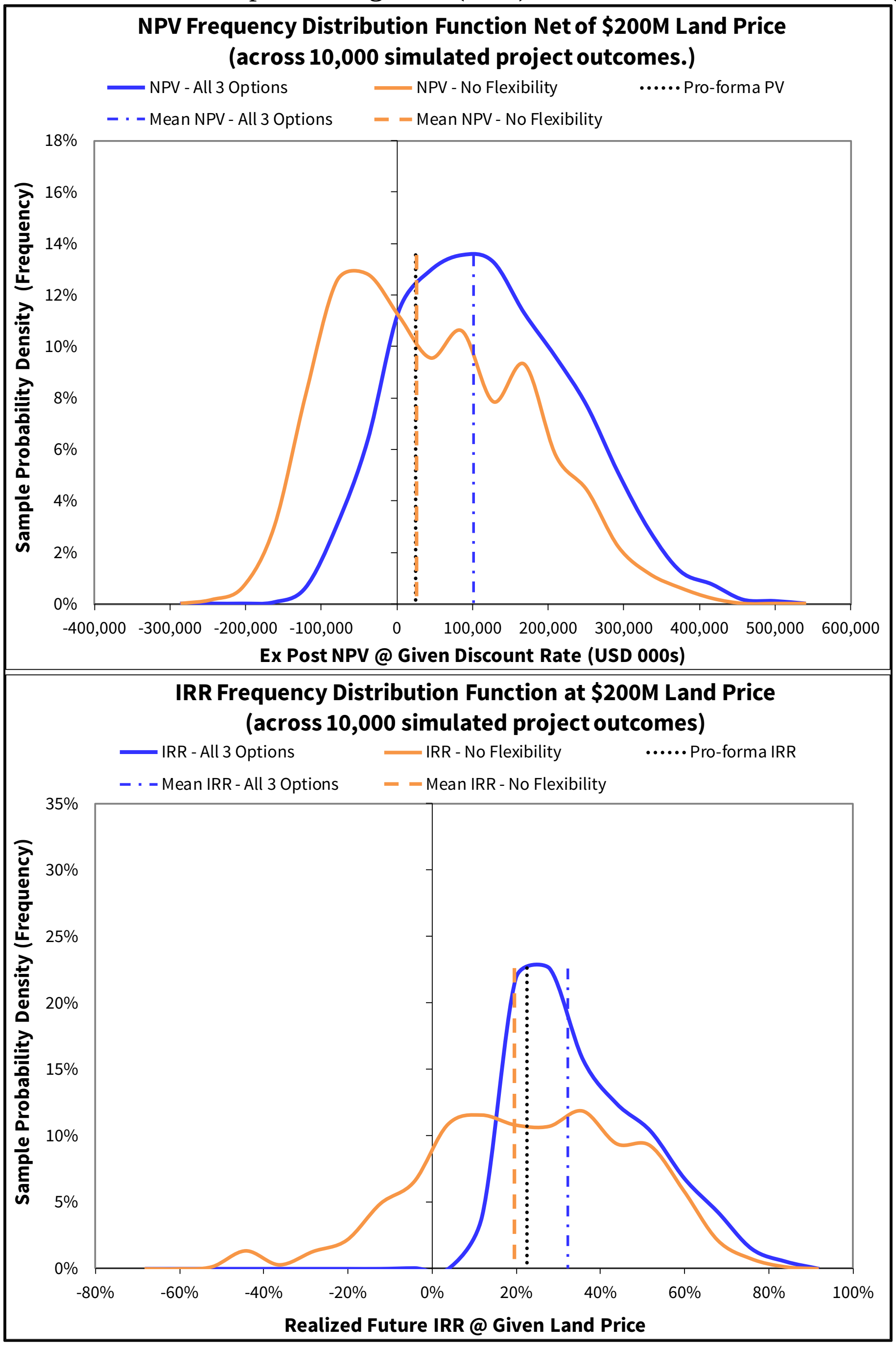
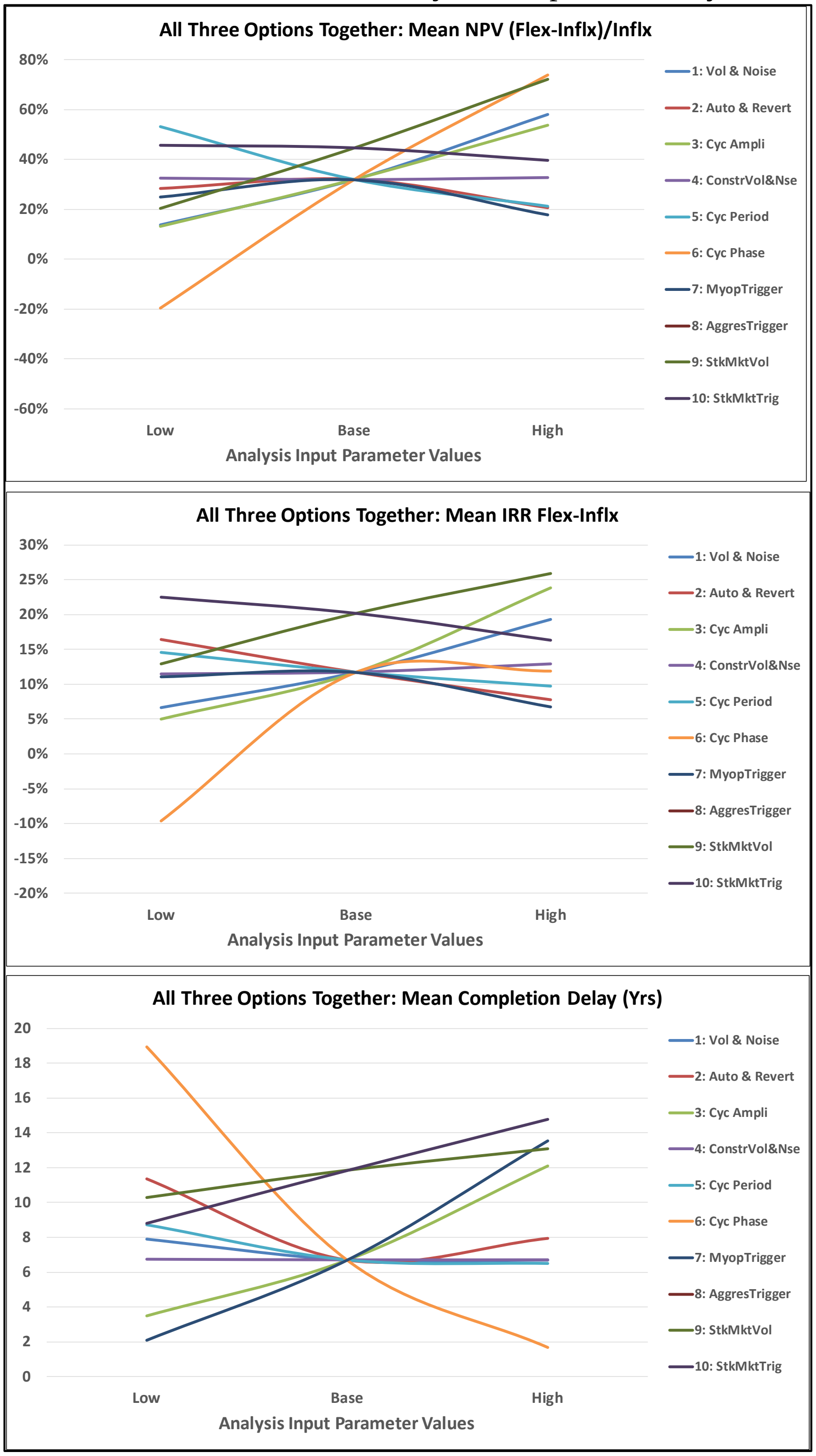


Exhibit 13:
Simulated Ex Post Mean Valuation & Return Effects of Three Development Options Separately and in Combination: Illustrative Vanilla Terrace Project

Simulated Ex Post Mean Performance Differential, Flexible Minus Inflexible:			
Performance Metric:	NPV Percent of Land Value	Internal Rate of Return	Mean Completion Delay (Yrs)
Start Delay Flexibility Only	21.5%	8.2%	1.9
Buildout Delay Flexibility Only	13.0%	7.8%	5.3
Switch Option Only	13.6%	5.8%	0.0
Start+BldOut Only	21.4%	9.0%	6.2
Start+Switch Only	33.8%	12.0%	1.5
BldOut+Swicth Only	25.7%	11.1%	4.1
All Three Options	33.9%	12.4%	4.8
Note: Buildout Delay Flexibility based on -20% trigger value. Start Delay based on zero trigger level.			

Exhibit 14:
Development Project Flexibility Value
Sensitivity Analysis of Various Price Dynamics and Decision Rule Assumptions
on the Mean Valuation, IRR, and Project Completion Delay Outcomes



Note: Base assumptions here include zero delay trigger for all timing options,
which is suboptimal for the subsequent buildout option.